Things I will remember about working with Jeff:

• Valued my work on accretion disk spectra, promoted it, included me in the effort, encouraged me, and advocated for me
• He was determined both to make it work and get it right
• He was always enthusiastic and wonderfully persuasive
Color-corrected Blackbodies

Integrate over radii with different temperature: a multitemperature blackbody:

$$T \propto r^{-3/4}$$

Essentially the DISKBB model (Mitsuda+ 1984)

Electron scattering and atomic opacity cause deviations from blackbody: sometimes approximated as a “color-corrected” blackbody (Shimura & Takhara, 1995):

$$I_\nu = \frac{1}{f^4} B_\nu (fT)$$
Disk Spectral Models

Self-consistent models of spectra at the disk surface must perform stellar atmospheres-like calculations:

- Solve for hydrostatic equilibrium
  \[- \frac{\partial P_{\text{tot}}}{\partial z} = \rho \Omega^2 z\]

- Solve for radiative equilibrium
  \[\nabla \cdot F = \epsilon\]

- Solve equations of radiative transfer and statistical equilibrium (with Compton scattering, Bremsstrahlung, and atomic opacities)

Solving large system of coupled PDE’s: typically involves iterative methods (complete linearization, accelerated lambda iteration)
Thin Disk Model Parameters

- $M$: black hole mass
- $L/L_{\text{edd}}$: luminosity/accretion rate
- $a_*$: black hole spin
- $\alpha$: stress parameter

Annuli Parameters

- surface density
- gravity ($g = \Omega^2 z$)
- effective temperature

Radial structure/emission
(Shakura & Sunyaev, 1973, Novikov & Thorne, 1973)

Vertical structure/radiative transfer
(TLUSTY, Hubeny & Lanz, 1995)

Photons follow geodesics
(KERRTRANS, Agol, 1997)

BHSPEC

- $i$: inclination

i
KERRBB vs BHSPEC

Similarities:
• Both models assume $F(R)$ based on Novikov-Thorne
  Both trace geodesics and include frequency shifts to get spectrum far from disk

Differences:
• BHSPEC uses spectrum computed using TLUSTY to obtain spectra at disk surface, ignores returning radiation
• KERRBB uses color-corrected blackbody, includes returning radiation

Which should we use to measure spin?

Jeff’s solution: both – fit BHSPEC with KERRBB to find $f$ and use with returning radiation in KERBB
Using multitemperature blackbody one finds $L \sim T^4$, where $T=T_{in}$.
Radius is nearly (but not exactly) constant

Using $f$

Use a color correction $f$ to “correct” the relations: $T_{eff} = T_{in}/f$

Implied radius is now constant!
Value of $f$ relatively unambiguous for models that look like color corrected blackbodies but not those that don’t look like color corrected blackbodies.
Parameters:

\[ Q : g = Q \, z \quad (= \Omega^2) \]

\[ T_{\text{eff}} \quad F = \sigma T_{\text{eff}}^4 \]

\[ m_0 = \Sigma / 2 \]

Graphs showing the relationship between log \( T_{\text{eff}} \) and \( \log Q \) with different values of \( m_0 \): 10^3, 10^4, 10^5 g/cm².

Legend:
- \( m_0 = 10^3 \) g/cm²
- \( m_0 = 10^4 \) g/cm²
- \( m_0 = 10^5 \) g/cm²
- \( m_0 = 316 \) g/cm²
- \( m_0 = 562 \) g/cm²
What happens if a disk doesn’t emit enough photons to match the energy that is being released by accretion?

It heats up!
Photon Starvation

What happens if a disk doesn’t emit enough photons to match the energy that is being released by accretion?

It heats up! Balance of photons and flux occurs for

\[ \eta_{ff}H = \sigma_{sb} T_{eff}^4 \]

with:

\[ H = \frac{\kappa_{es}\sigma_{sb} T_{eff}^4}{cQ} \quad T \approx (\kappa_{es}m_0)^{1/4} T_{eff} \quad \rho \approx \frac{m_0}{H} \]

Gives:

\[ Q = \frac{m_p^{7/8} \kappa_{es} \sigma_{sb}^2 T^{7.5}}{\eta_0 c m_0^{2.125}} \]
Spectral hardening factors only get large when disk becomes photon starved!
Soft X-ray excess
For supermassive black holes with \( M \sim 10^6 \, M_{\odot} \), emission from inner accretion disk might reach soft X-ray, but soft excess would seem to require larger \( f \) than \( \alpha \)-disk models predict.

Low/hard state X-ray binaries
Some interpretations of low/hard spectral states require disk has \( f > 2 \) (see e.g. Reynolds & Miller, 2013; Salvesen+ 2013)
Summary

• We use TLUSTY models to explore variation of spectral hardening over large range of parameters
• We find that $f \sim 1.4 - 2$ over the range of accretion rates and masses in X-ray binaries
• AGN are expected to have larger $f$ than X-ray binaries at the same accretion rate.
• Large values of $f (>2)$ are found when disks become photon starved, which happens at accretion rates below Eddington for mass surface densities below about 1000 g/cm$^2$
Estimating $f$ for individual Annuli

No single way to fit a color corrected blackbody to our models.

Could include:
- Absorption
- Weight by counts
- Instrument response

Red: peak below 0.5 keV
Blue: peak above 0.5 keV
Merloni, Fabian, and Ross (2000) concluded that putting a fraction of dissipation in corona would make the disk spectrum harder. Our results go the opposite way. Taking dissipation out of the disk makes it colder, which reduces $f$!