Separate universes and large-scale clustering

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Cosmological structure from inflation

\[ \ln (1/k) \]

- **super-horizon**
- **sub-horizon**

- **inflation**
  - **RD**
  - **MD**

- **CMB**

- **Large-scale structure**

- **\( 1/(aH) \)**

- **\( \ln a(t) \)**

- **curvature perturbation \( \zeta \)**
- **density perturbation \( \delta \)**
Primordial non-gaussianity

Departure from Gaussian statistics reveals physics beyond single-field inflation
(1) nonlinear coupling (negligible in minimal model)
(2) new degrees of freedom

\[
\left\langle \phi_{\text{ini}}(k_1) \phi_{\text{ini}}(k_2) \phi_{\text{ini}}(k_3) \right\rangle = B_{\phi\phi\phi}(k_1, k_3) (2\pi)^3 \delta_D(k_1 + k_2 + k_3)
\]

- **squeezed**
  \[ k_L \ll k_S \quad k_1 \approx -k_2 \approx k_S \]
  \[ B_{\phi\phi\phi}(k_L, k_S) \sim f_{\text{NL}} P_\phi(k_S) P_\phi(k_L) \]
  \( f_{\text{NL}} = 0.8 \pm 5.0 \)

- **equilateral**
  \[ k_3 = k_L \]
  \[ B_{\phi\phi\phi}(k_L, k_S) \sim f_{\text{NL}} \frac{k_L^2}{k_S^2} P_\phi(k_S) P_\phi(k_L) \]
  \( f_{\text{NL}} = -4 \pm 43 \)

- **folded**
  \[ B_{\phi\phi\phi}(k_L, k_S) \sim f_{\text{NL}} \frac{k_L^2}{k_S^2} P_\phi(k_S) P_\phi(k_L) \]
  \( f_{\text{NL}} = -26 \pm 21 \)

CMB constraint from Planck 2015

Need \( \sigma(f_{\text{NL}}) \approx O(1) \) to distinguish between models (Alvarez et al 2014)
Probing non-gaussianity with large-scale structure

Dark Matter or galaxy perturbation trace primordial perturbation

\[ \delta (\vec{x}, t) = \rho (\vec{x}, t) / \bar{\rho}(t) - 1 \]

We can measure LSS bispectrum:

\[ \left\langle \delta (\vec{k}_1, z) \delta (\vec{k}_2, z) \delta (\vec{k}_3, z) \right\rangle \]

Primordial NG imprinted

End of inflation

clustering under (nonlinear) gravity

source redshift

projection: RSD, lensing, Sachs-Wolfe, etc.

present
Galaxies are biased tracers of DM

\[ \delta_g = b_L \delta + (b_L - 1) f_{NL} \phi \]

(Dalal, Dore, Huterer & Shirokov 2008)
Separate universe: spherical top-hat overdensity

average universe

over-dense patch

Uniform top-hat matter + CC

Local universe, over-dense with positive curvature

Valid in full general relativity!
Conformal Fermi Coordinates (CFC)

Einstein’s elevator
Fermi Normal Coordinates (FNC)  (Manasse & Misner 1963)

Coarse-grain first!

\[(g_F)_{\mu\nu} (\tau_F, x_F^i) = a_F^2 (\tau_F) \left( -\eta_{\mu\nu} + \mathcal{O} \left[ (x_F^i)^2 \right] \right)\]
Separate universe in CFC  

(LD, Pajer & Schmidt 2015)

The usual FNC

homogeneous expansion

\[ ds^2 = - \left[ 1 - \left( \dot{H} + H^2 \right) r^2 \right] dt^2 + \left[ 1 - \frac{1}{2} \left( H^2 + \frac{K}{a^2} \right) r^2 \right] \delta_{ij} dx^i dx^j \]

+ Newtonian tidal terms \( \sim \partial \partial \Phi, \partial \cdot V \)

+ General relativistic corrections \( \sim \ddot{\Phi}, H \dot{\Phi}, H \partial \Phi \)

ALL ``GR corrections'' absorbed into \( a_F, H_F \) and \( K_F \)

\[ ds^2 = a_F^2 (\tau_F) \left[ - \left( 1 + r_F^2 \left( \partial_k \partial_l - \frac{1}{3} \delta_{kl} \partial^2 \right) \Phi \right) d\tau_F^2 + \left( 1 - r_F^2 \left( \partial_k \partial_l - \frac{1}{3} \delta_{kl} \partial^2 \right) \Psi \right) \frac{\delta_{ij} dx_F^i dx_F^j}{\left( 1 + K_F r_F^2 / 4 \right)^2} \]  

Advantages compared to FNC:

• Ability to extrapolate to (super-)horizon scales
• Physical definition of local expansion rate
Conditions for separate universe

- Local curvature

Consider multiple fluids \( I = 1, 2, 3, \ldots \)

\[ K_F = \frac{2}{3} \partial^2 \mathcal{R} \]

\[ \frac{dK_F}{dT} = -\mathcal{H}^2 \left[ \sum_I (1 + w_I) \Omega_I \partial^2 (V_I - V_o) \right] + \frac{2}{3} \mathcal{H} \partial^2 (\Phi - \Psi) \]

- Exact “local universe” conditions (always true for matter + CC):
  - NO anisotropic stress
  - ALL fluids co-move along geodesics (free-falling)
  - Non-adiabatic pressure allowed

- Approximate conditions:
  - Sound horizons small
  - “Dark energy” component \( w \approx -1 \)
Calculating in CFC: squeezed matter bispectrum

First add small-scale perturbations

\[ ds^2 = a_F^2(\tau_F) \left\{ - \left[ 1 + 2\phi + \left( \partial_k \partial_l \Phi - \frac{1}{3} \delta_{kl} \partial^2 \Phi \right) x^k x^l \right] d\tau_F^2 \\
+ \left[ 1 - 2\psi - \left( \partial_k \partial_l \Psi - \frac{1}{3} \delta_{kl} \partial^2 \Psi \right) x^k x^l \right] \frac{\delta_{ij} dx^i dx^j}{(1 + K_F r^2/4)^2} \right\} \]

Usual Euler-Poisson system

\[ \partial^2 \phi = \frac{3}{2} \mathcal{H}_F^2 \Omega_m^F \delta \]

\[ \delta' + \partial_i \left[ (1 + \delta) v^i \right] = 0 \]

\[ v_i' + \mathcal{H}_F v_i + v^j \partial_j v_i + \partial_i \phi = -K_{ij}^\Phi x^j \]

Formally identical to Newtonian, but extrapolate to \( k_L < aH \):

• equal proper time
• proper wavelength

\[ \langle \delta(k_1) \delta(k_2) \Delta_{sc}(k_L) \rangle' = \left[ \frac{26}{21} + \left( \mu_{SL}^2 - \frac{1}{3} \right) \left( \frac{8}{7} - \frac{d \ln P_\delta(k_S)}{d \ln k_S} \right) \right] P_{sc}^\Delta(k_L) P_\delta(k_S) \]
A previous confusion is resolved

Scale-dependent biasing from general relativity?

\[ f_{NL}^{local} = -\frac{5}{3} \]

Argument: nonlinear relation between initial density and initial curvature in full GR.

What was overlooked: (LD, Pajer & Schmidt 2015; de Putter, Dore & Green 2015) short-wavelength is not measured in proper units; a long-wavelength metric perturbation modulates local proper distance measure.

NO scale-dependent bias from GR (Baldauf, Seljak, Senatore & Zaldarriaga 2015)

NO scale-dependence in halo/galaxy shape correlation induced by GR (Schmidt, Chisari & Dvorkins 2015)

Or any other correlation in halo/galaxy properties at linear order in metric
Projection effects

Apparent, scale-dependent bias

\[ \Delta b (k_L) \sim \frac{\phi}{\delta} \sim \frac{(aH)^2}{k_L^2} \]

Assume no physical clustering

Source redshift \( z \)

(e.g. Jeong, Schmidt & Hirata 2012, Camera, Santos & Maartens 2015)
Overview of large-scale clustering

apparent, scale-dependent bias

\[ \Delta b(k_L) \sim \frac{\phi}{\delta} \sim \frac{(aH)^2}{k_L^2} \]

Nonlinear gravitational clustering: scale-independent bias

\[ \Delta b(k_L) \sim (b - 1)\delta_c \frac{(aH)^2}{k_L^2} \]

intrinsic, scale-dependent bias

CAVEAT: projection can also be tracer dependent --- magnification bias, evolution bias, etc.

Genuine local-type NG

observer
Galaxy clustering on large scales is a promising probe of (local-type) primordial non-gaussianity. $\sigma(f_{NL}) \sim O(1)$ requires accurate description in general relativity.

We construct CFC in which local dynamical effects of a long-wavelength perturbation on structure formation is isolated. In full general relativity, the effect is equivalent to a modified expansion rate and curvature, plus a tidal force.

We disprove a scale-dependent biasing caused by general relativistic dynamics. Detection of nonzero scale-dependent bias, if not explained by relativistic projection effects, is a smoking gun of new physics beyond single-field inflation.