Stumbling Towards Simulations of Collisionless Black Hole Accretion Flows

Kyle Parfrey

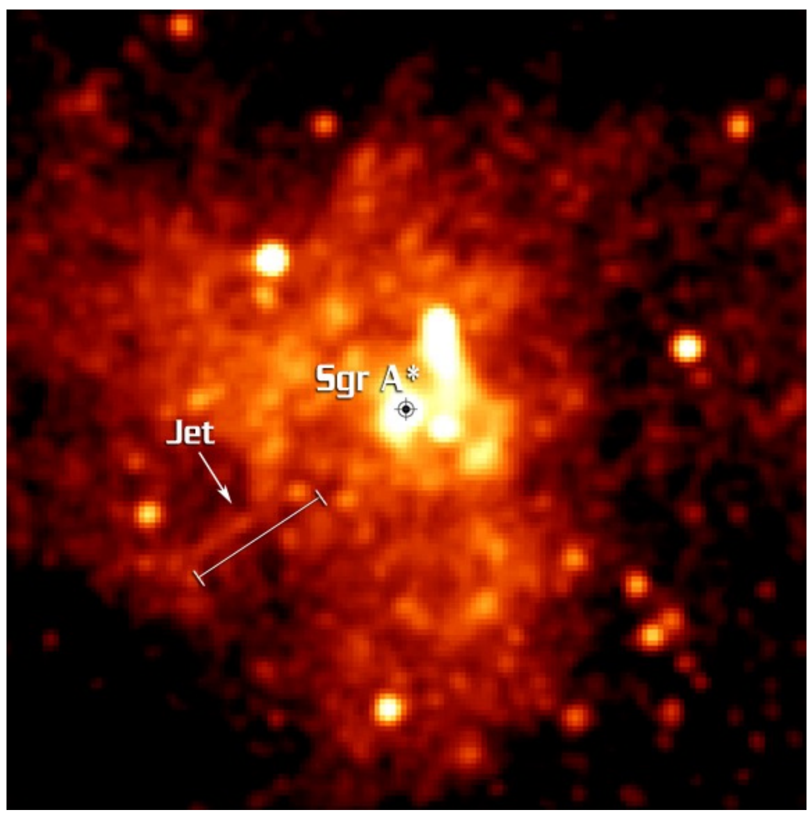
Lawrence Berkeley
National Laboratory

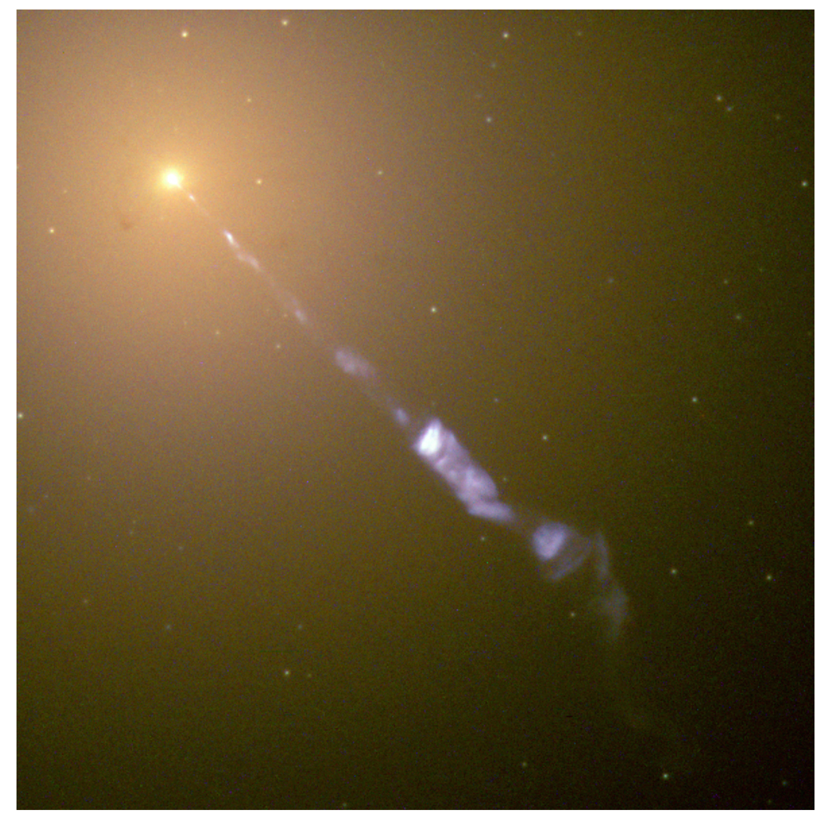
Einstein Fellows Symposium, October 18, 2016

mm VLBI imaging of accretion flows on horizon scales

Sgr A*

Chandra





M87 jet

Hubble

$r_g = GM/c^2$	0.05 AU	30 AU
apparent size	5 μas	2 μas

Event Horizon Telescope

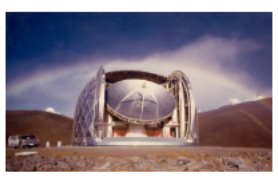
Current

Arizona Radio Observatory/Submillimeter-wave Astronomy (ARO/SMT)

Atacama Pathfinder EXperiment (APEX)

Atacama Submillimeter Telescope Experiment (ASTE)

Combined Array for Research in Millimeter-wave Astronomy (CARMA)

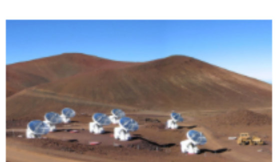


Caltech Submillimeter Observatory (CSO)

Institut de Radioastronomie Millimetrique (IRAM) 30m

James Clerk Maxwell Telescope (JCMT)

The L arge Millimeter Telescope (LMT)



The Submillimeter Array (SMA)

Now

1.3 mm — 23 μas

Soon

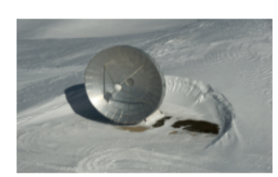
0.87 mm — 15 μas

Eventually?

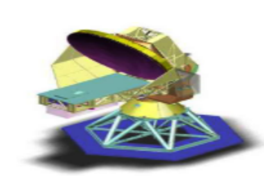
0.65 mm — 11 μas

Future

Atacama Large Millimeter/submillimeter Array (ALMA)

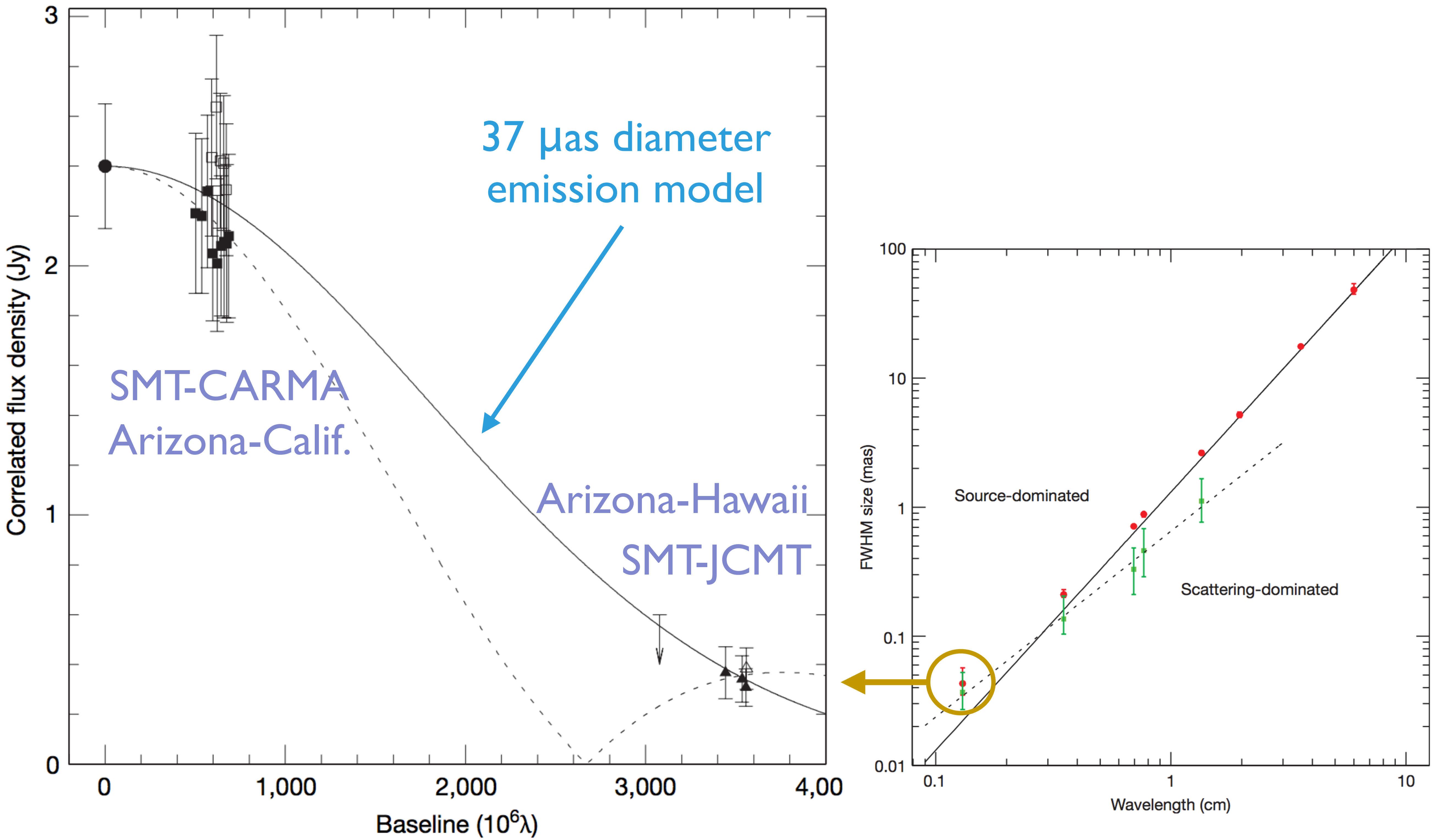


Plateau de Bure interferometer



South Pole Telescope

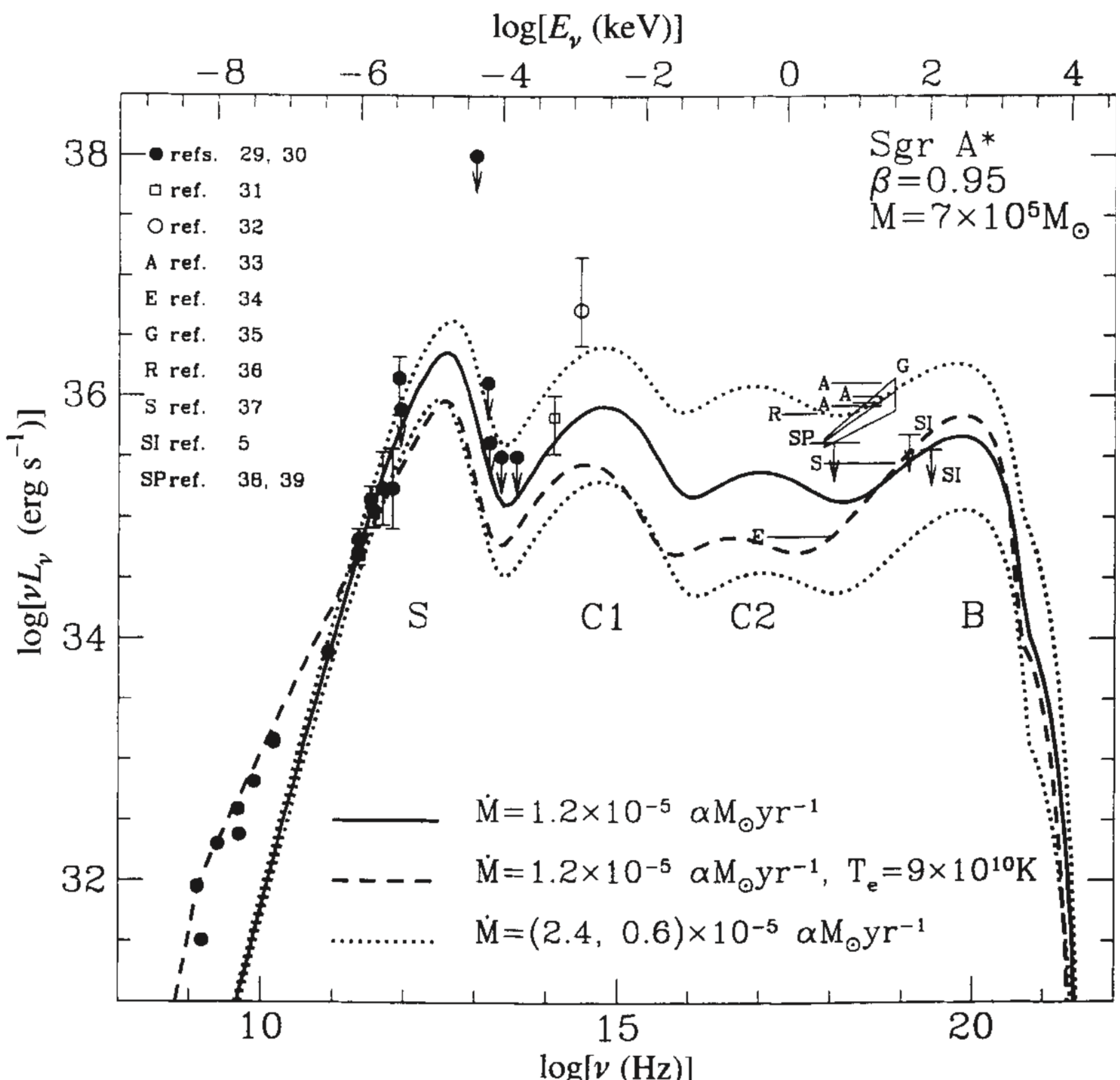
Sgr A* size measurement at 1.3mm

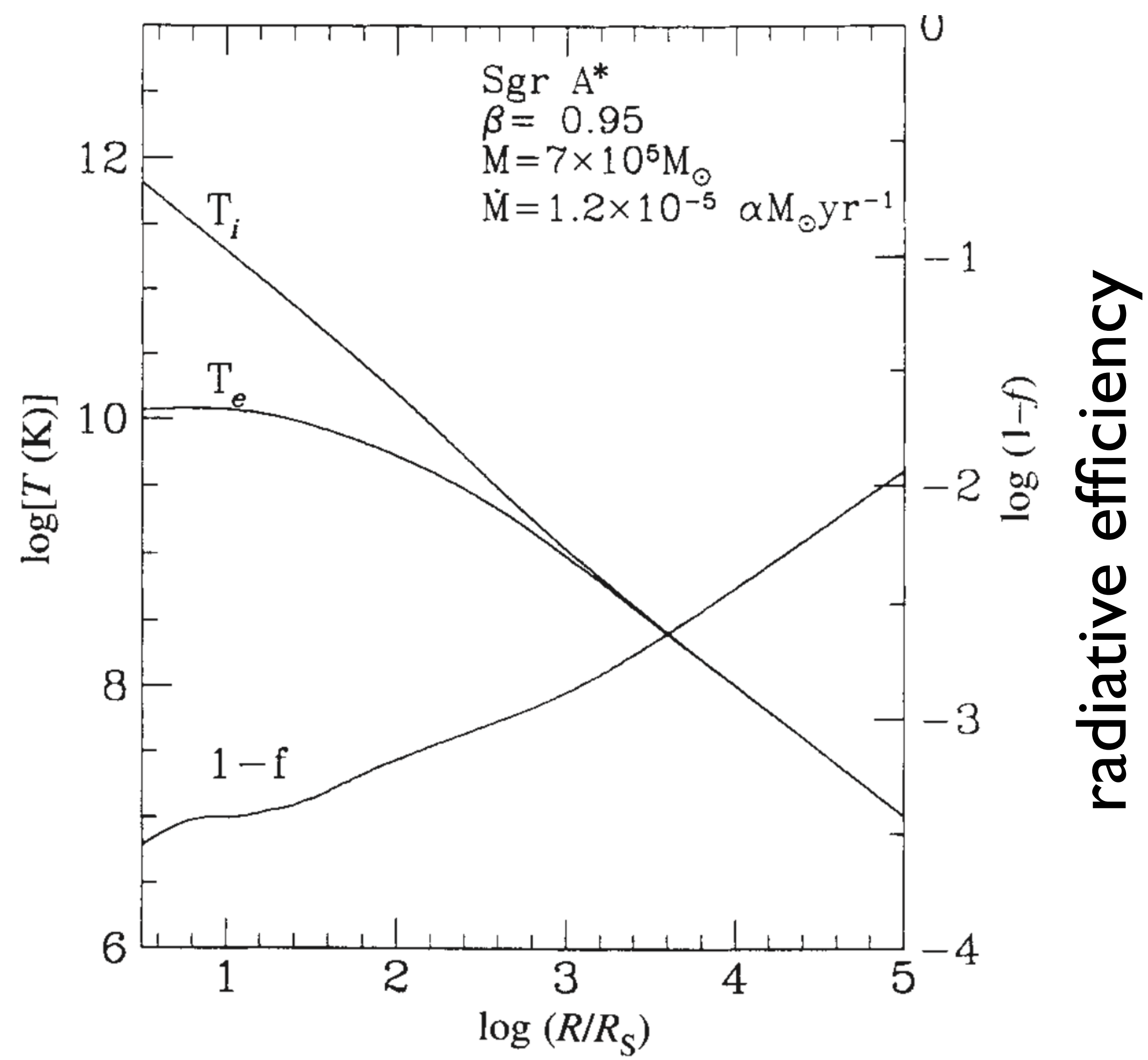


Doeleman+ 2008

These flows expected to be *collisionless*

Coulomb mean free path ≫ system scales ~ r_g





Narayan+ 1995

fit of ADAF model to Sgr A*

Collisionless accretion

At low accretion rate, flow can have ***low-density*** structure which:

1. Can't cool → hot & ∴ geometrically thick
2. Optically thin
3. All collision timescales longer than accretion timescale

↓

e^- & protons have different T,
and generally non-thermal distribution

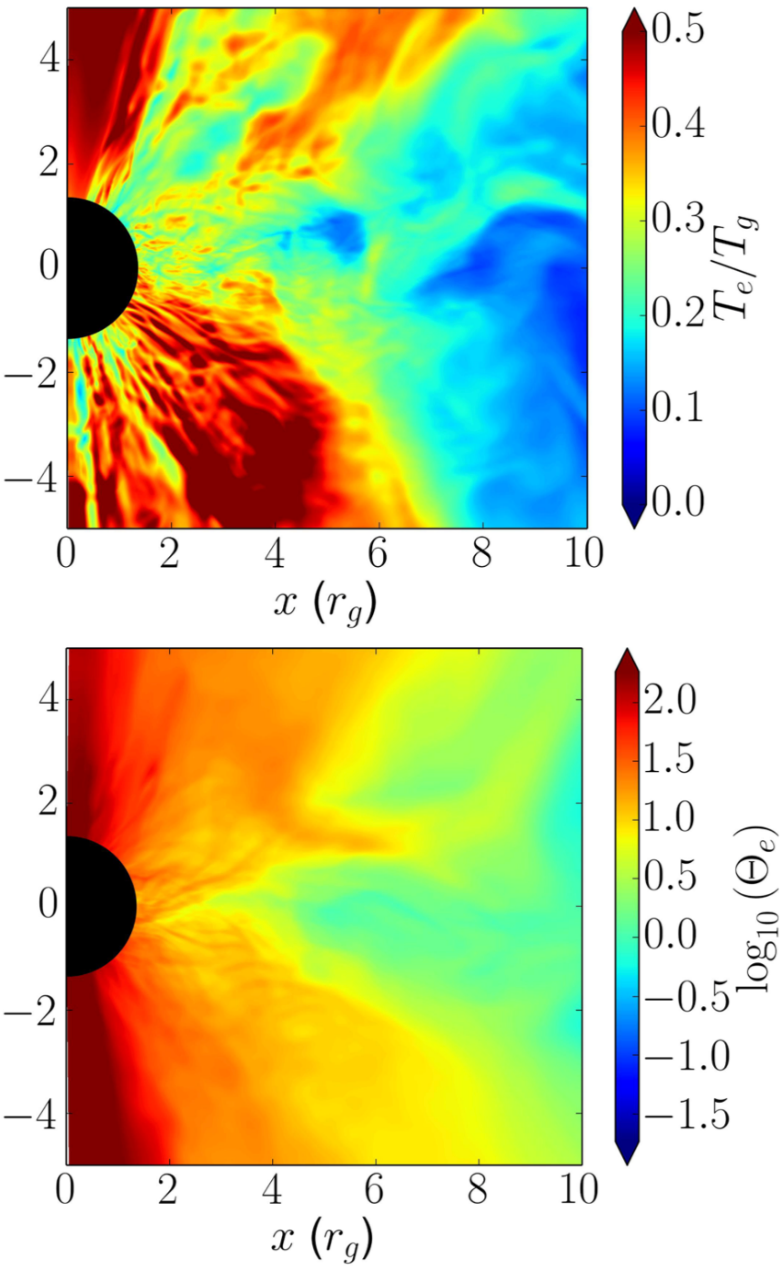
Sgr A*, M87, many low-L AGN, X-ray binaries in hard state…

+ other relativistic collisionless flows: e.g. ***coronae*** of thin discs

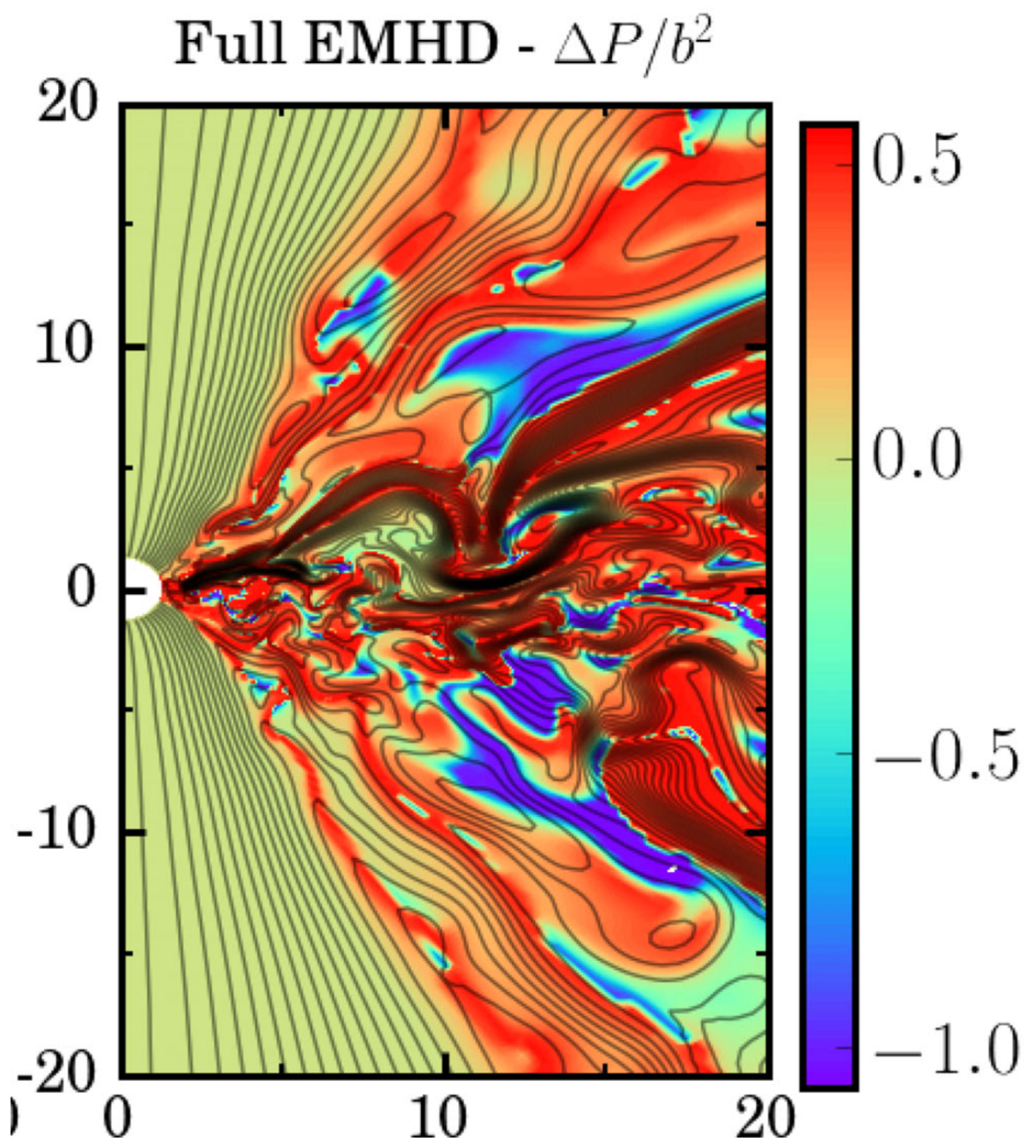
Standard ideal/resistive MHD assumptions don't hold

Extended fluid models for BH accretion

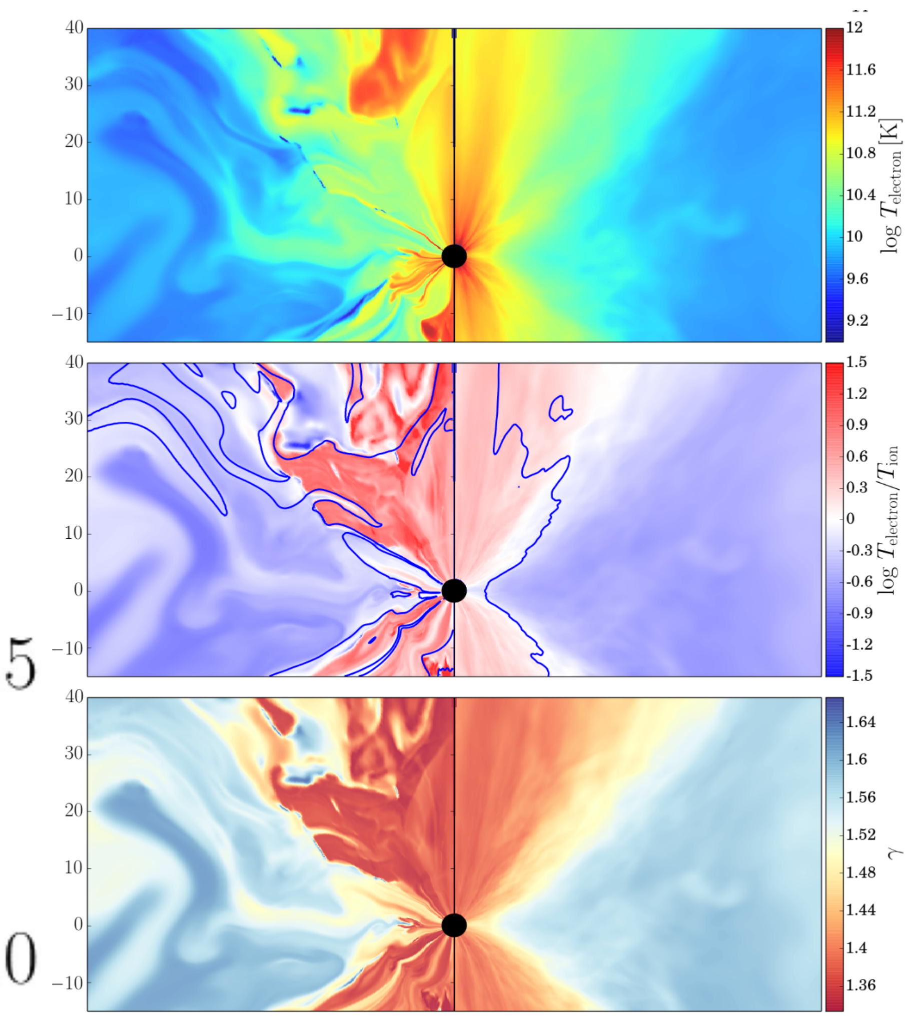
Add e⁻ thermodynamics, anisotropic pressure, …



Ressler+ 2015



Foucart+ 2016



Sadowski+ 2016

Still necessarily collisional physics

Collisionless physics: plasma kinetics

$$\partial_t \boldsymbol{E} = \nabla \times \boldsymbol{B} - \boldsymbol{J}$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

Maxwell's equations

$$\nabla \cdot \boldsymbol{E} = \rho_\mathrm{e}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

+

(a) Continuum dynamics

$$\frac{\partial f_\mathrm{s}}{\partial t} + \boldsymbol{v}_\mathrm{s} \cdot \nabla f_\mathrm{s} + q_\mathrm{s} \left(\boldsymbol{E} + \boldsymbol{v}_\mathrm{s} \times \boldsymbol{B} \right) \cdot \frac{\partial f_\mathrm{s}}{\partial \boldsymbol{p}} = 0$$

s : electrons, ions

Solve for distribution function $f(\boldsymbol{x},\boldsymbol{p},t)$

—Vlasov-Maxwell system

or

(b) Particle dynamics

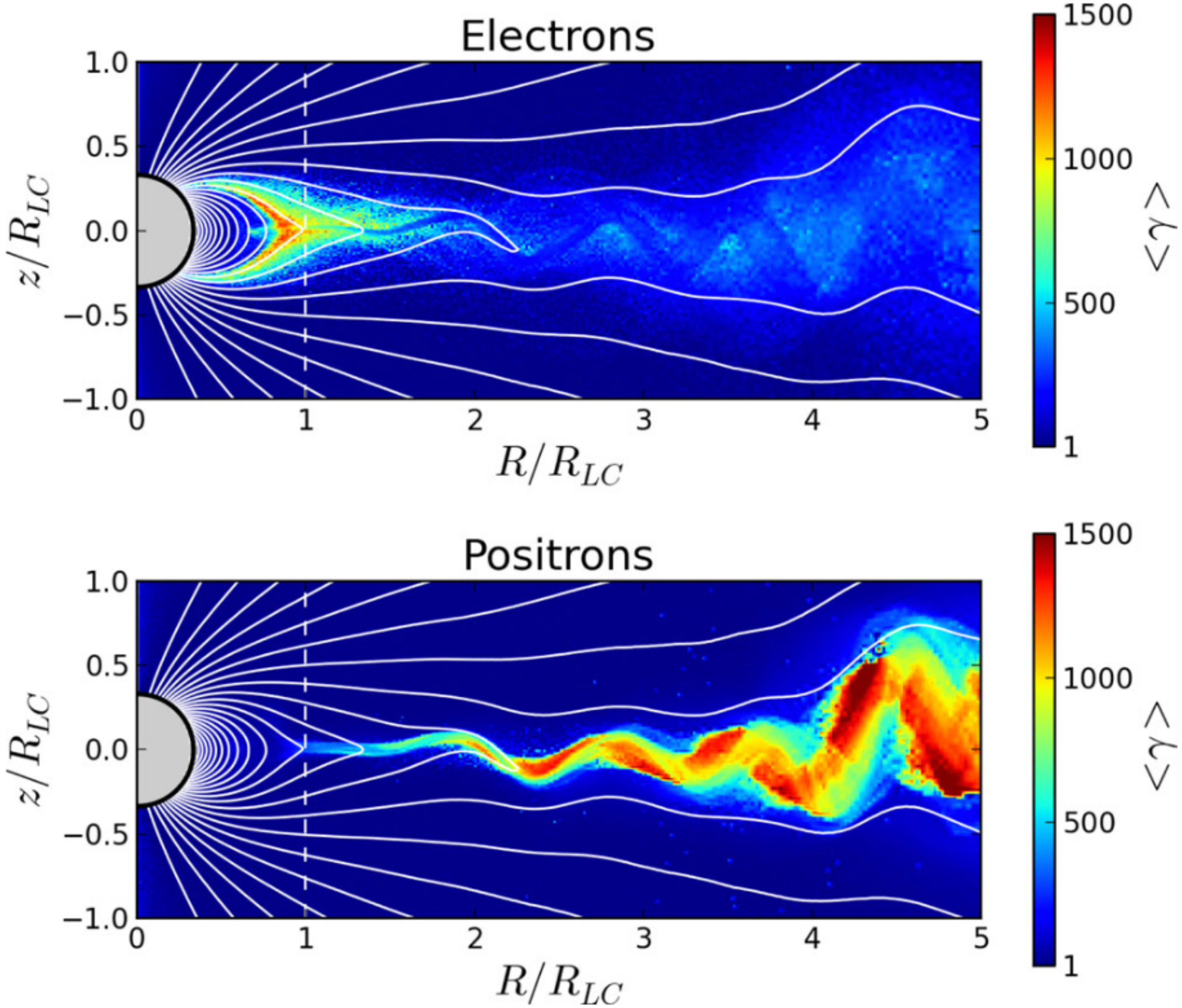
$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = q_i \left(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B} \right) \qquad \frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

$i = 0, \ldots, N$: particles

Solve for 6 fields: E, B + (3D momentum space or N particles)

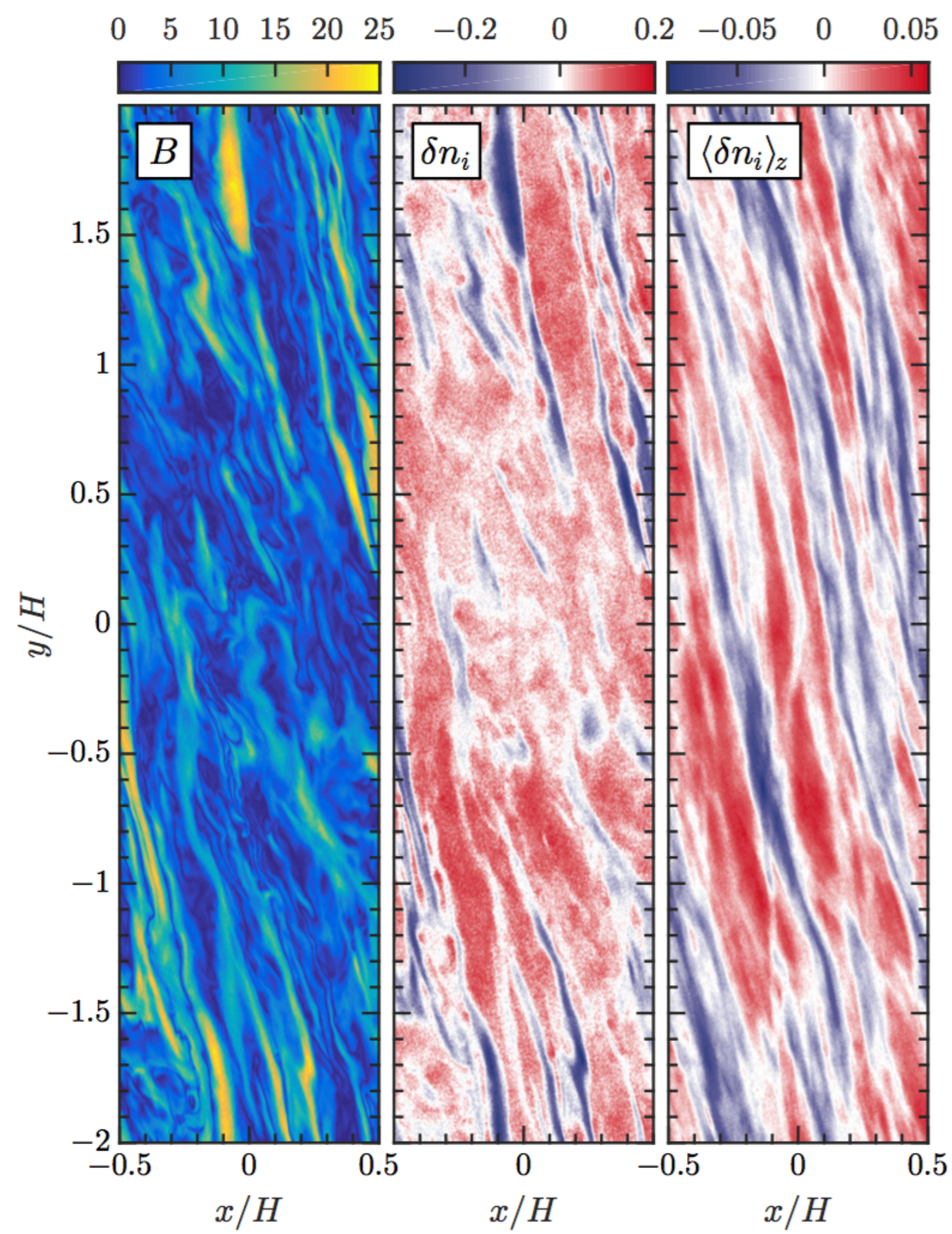
Particle-based simulations in action

Global pulsar magnetosphere



Cerutti+ 2015

Shearing-box MRI (hybrid)



Kunz+ 2016

PIC sims 1. – fields

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

3+1 ADM form

$$\nabla \cdot \boldsymbol{D} = \rho_{\mathrm{e}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

curved spacetime acts like nonlinear material

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B}$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D}$$

Komissarov 2004

particles determine current density $\boldsymbol{j}$, then

$$\boldsymbol{J} = \alpha \boldsymbol{j} - \rho \boldsymbol{\beta}$$

$\alpha, \boldsymbol{\beta}$: known functions coming from 4-metric

PIC sims 2. – particles

Start from Hamiltonian: $H = \pi_i v^i - L$

conjugate momentum $\pi_i = p_i + qA_i$

with kinetic momentum p_i and $v^i = \frac{\mathrm{dx^i}}{\mathrm{dt}}$

Lagrangian $L = -m\alpha/\Gamma + qA_j v^j - qA_t$

PIC sims 2. – particles

Start from Hamiltonian: $H = \pi_i v^i - L$

conjugate momentum $\pi_i = p_i + qA_i$

with kinetic momentum p_i and $v^i = \dfrac{\mathrm{dx^i}}{\mathrm{dt}}$

Lagrangian $L = -m\alpha/\Gamma + qA_j v^j - qA_t$

Hamilton's equations give

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{\alpha}{m\Gamma}p^i - \beta^i$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -m\Gamma\partial_i\alpha + p_j\partial_i\beta^j - \frac{\alpha}{2\Gamma m}\partial_i(\gamma^{lm})p_l p_m + q\left\{\alpha D_i + \epsilon_{ijk}(v^j + \beta^j)B^k\right\}$$

gravitational acceleration

~ extrinsic curvature

Lorentz force

Parfrey & Quataert, in prep

How should you solve these things?

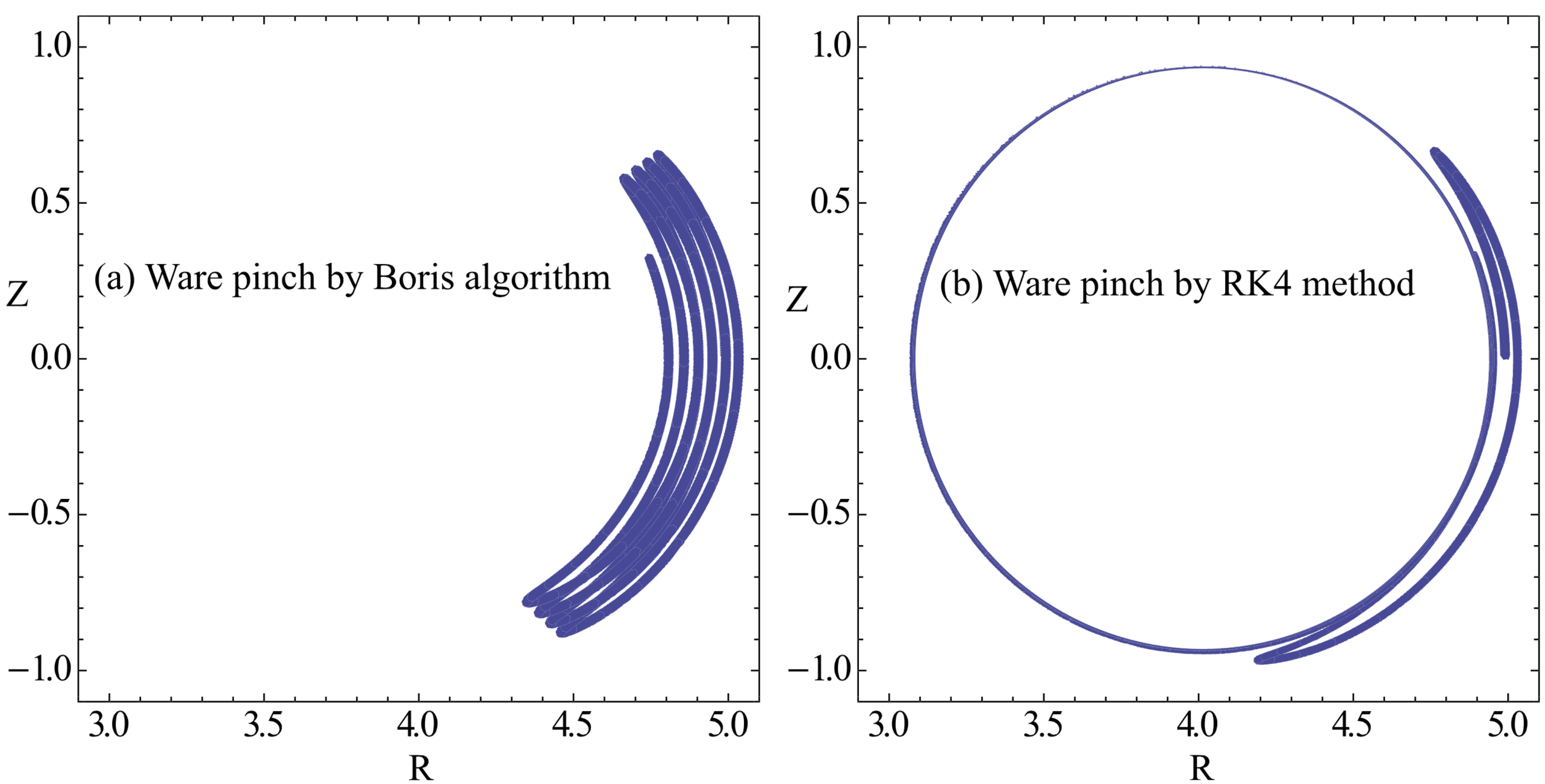
Ideally want *symplectic* integrator

preserves symplectic two-form: $s_{\mu\nu} = x_\mu \wedge p_\nu$

very good energy stability

Plasma physicists use ***Boris push*** for Lorentz force

not symplectic, but volume preserving: $|x_\mu \wedge p_\nu|$ is maintained



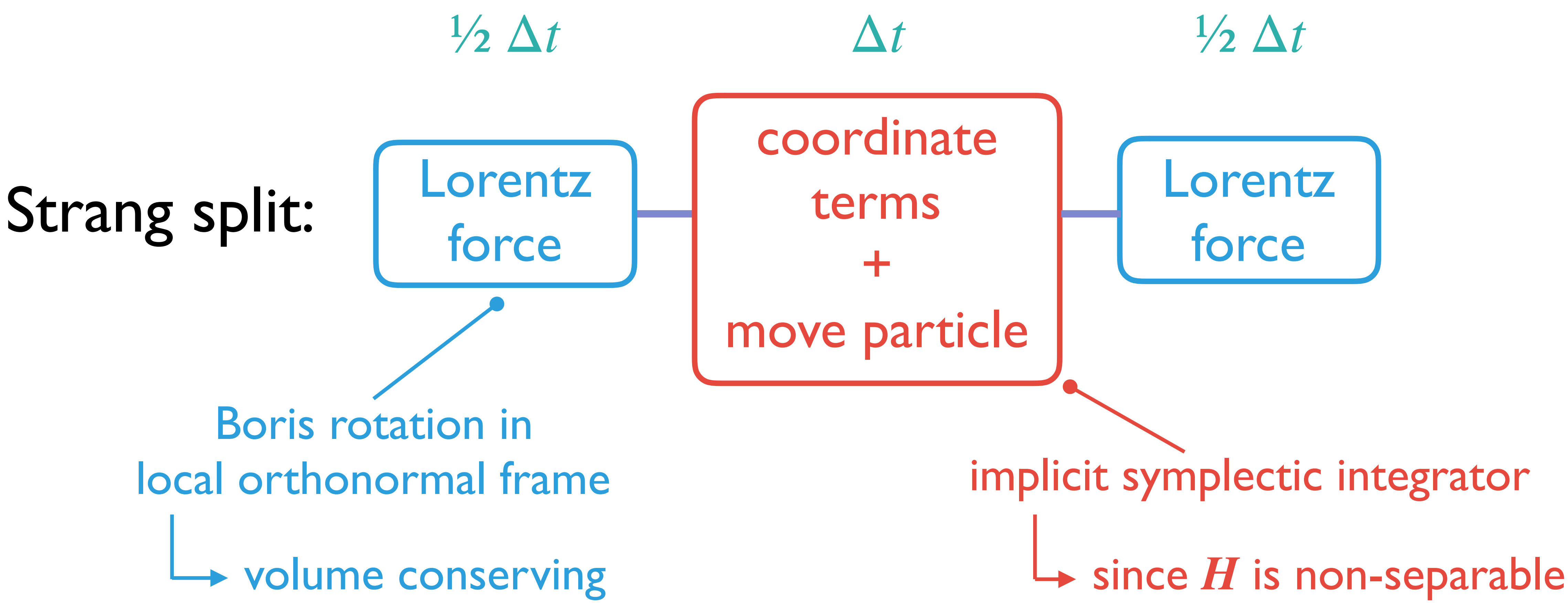
Qin+ 2013

"Why is Boris Algorithm So Good?"

Particle integrator scheme

Requirements:

1. conserve phase-space volume, $|x \wedge p|$
2. time-symmetric



$$\Gamma = \sqrt{1 + \gamma^{ij} p_i p_j / m^2}$$

fuses spatial & momentum dependence everywhere

Simple test

Flat spacetime: $\alpha = 1, \beta^i = 0$

Uniform magnetic field in *z*-direction…

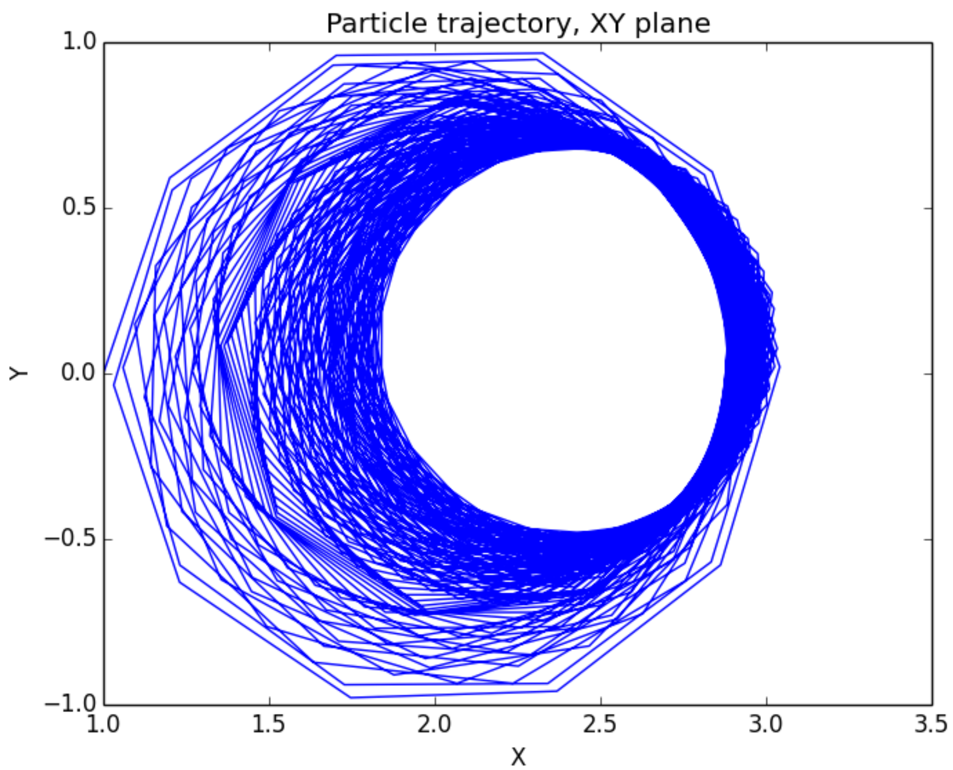
…but solve entirely in spherical coordinates

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\frac{1}{2\Gamma m}\partial_i(\gamma^{lm})p_l p_m + q\left\{ D_i + \epsilon_{ijk} v^j B^k \right\}$$

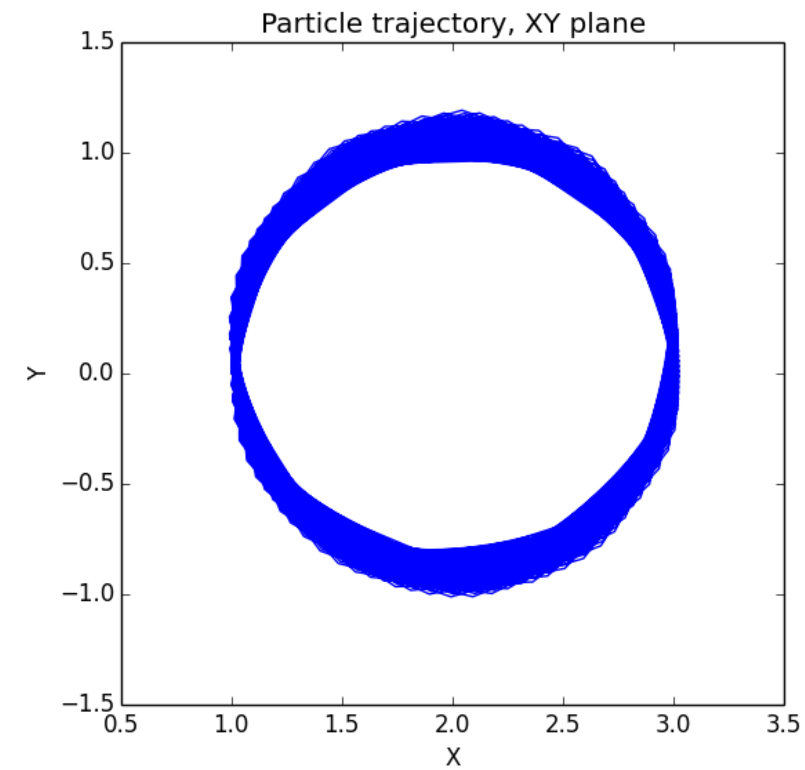
1. Non-trivial coordinate-force term
2. Local orthonormal frames (for Boris pushes) vary spatially

i.e. $B_{\hat{r}},\ B_{\hat{\theta}},\ B_{\hat{\phi}}$

Uniform *B* test

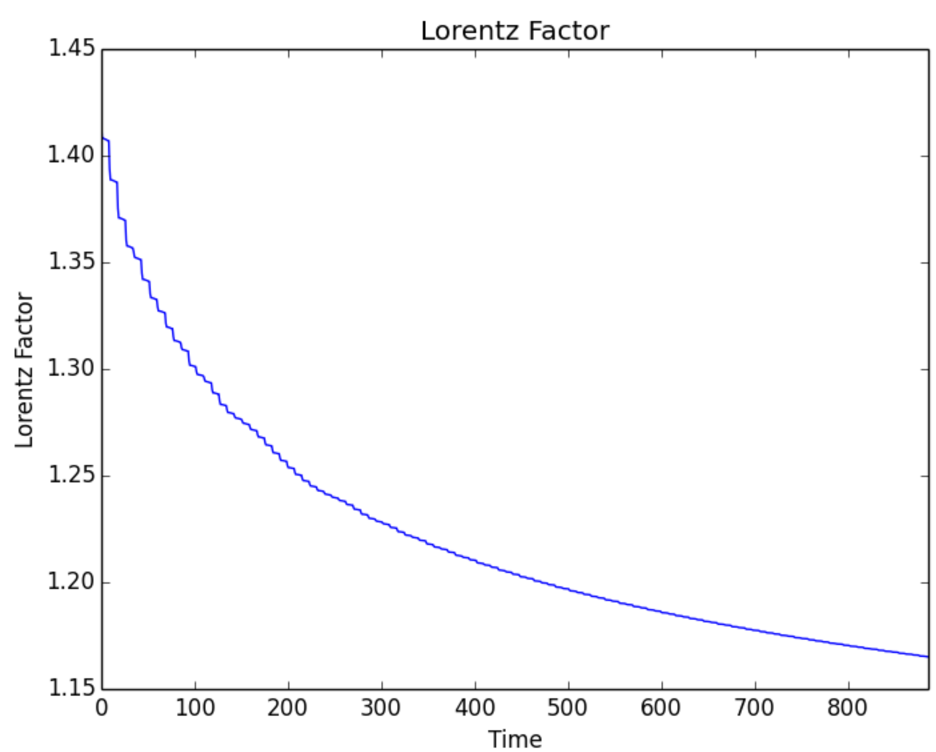


trajectory

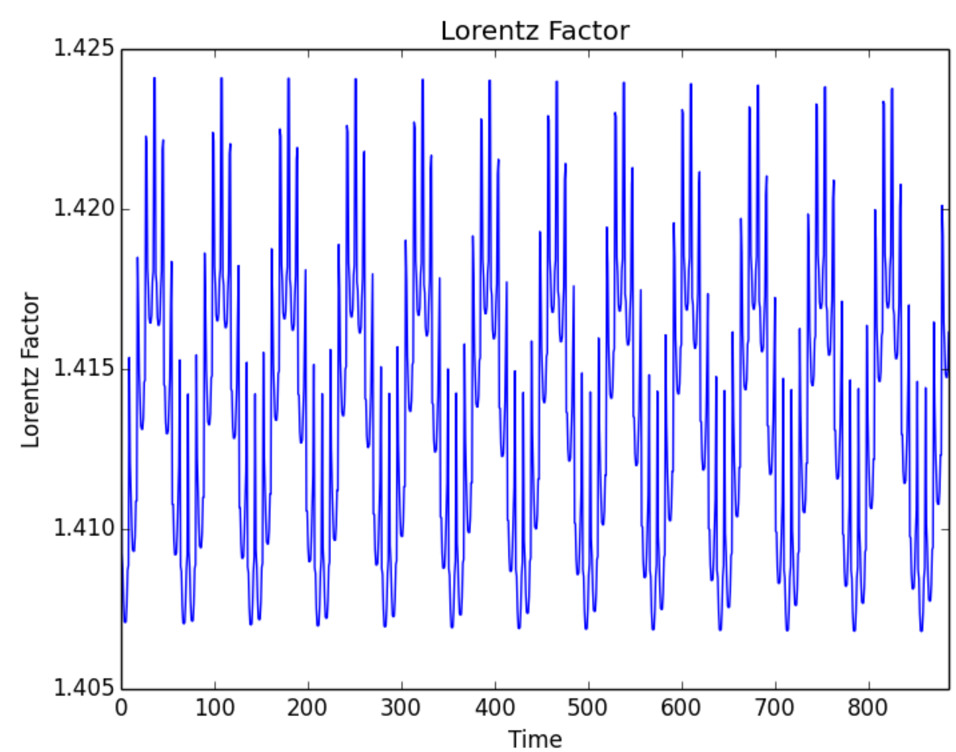


3rd order
Runge-Kutta

2nd order
implicit midpoint



Lorentz factor



Slightly more complicated test

$$B_z \propto R$$

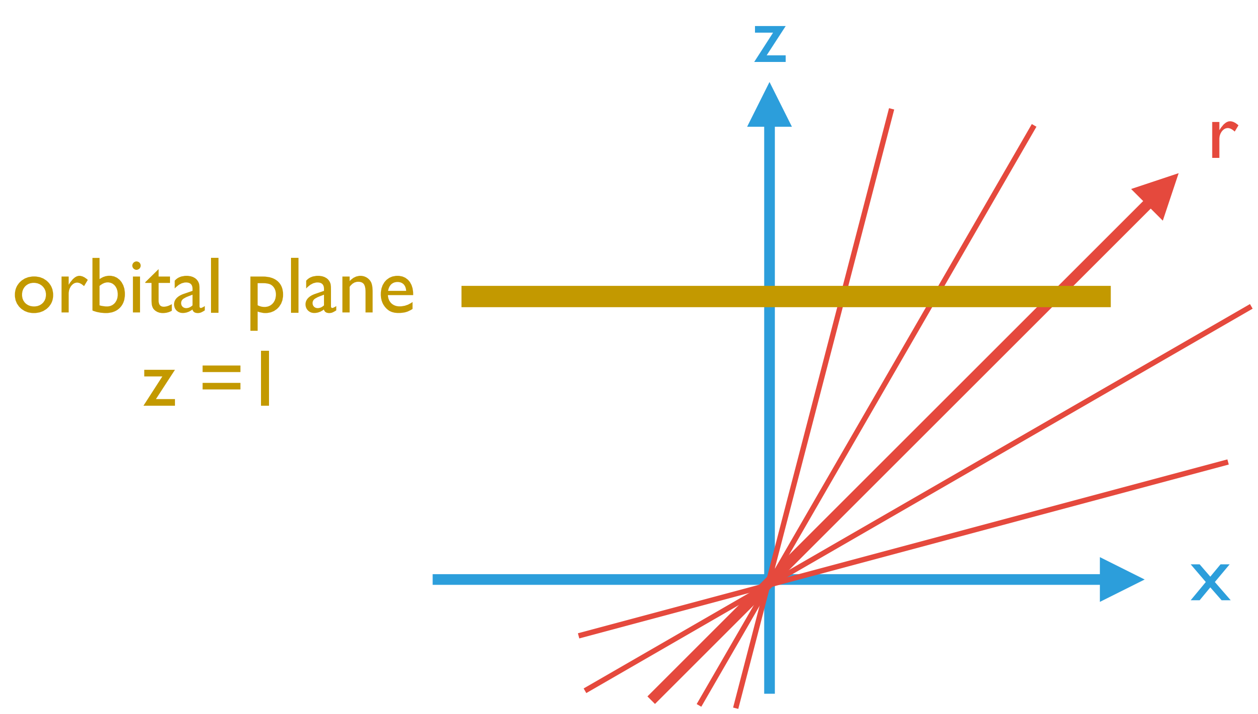
now have $\nabla \boldsymbol{B}$ & $\boldsymbol{E}\times\boldsymbol{B}$ drifts

$$E_R \propto R^{-2}$$

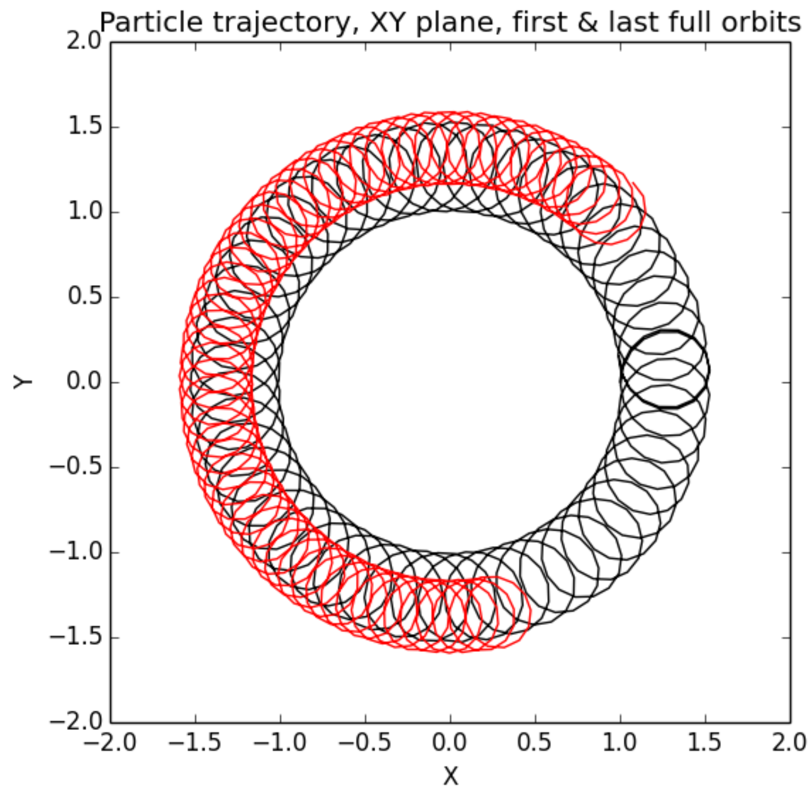
and raise plane of Larmor orbit off the equator

↓

now the coord. system isn't symmetric above/below the orbital plane

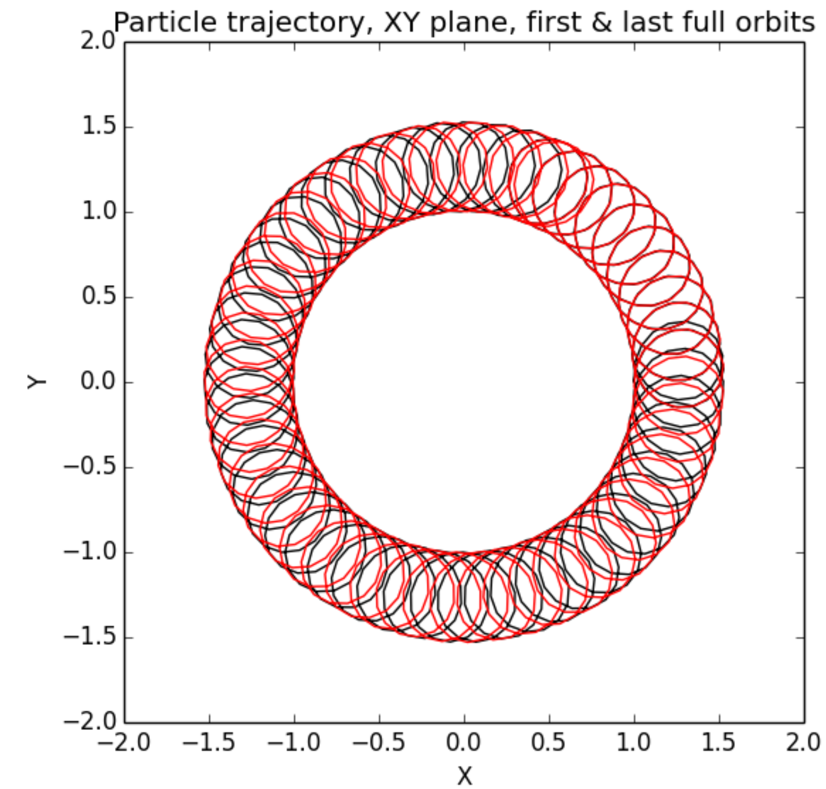


Balancing ∇B & $E\times B$ drifts



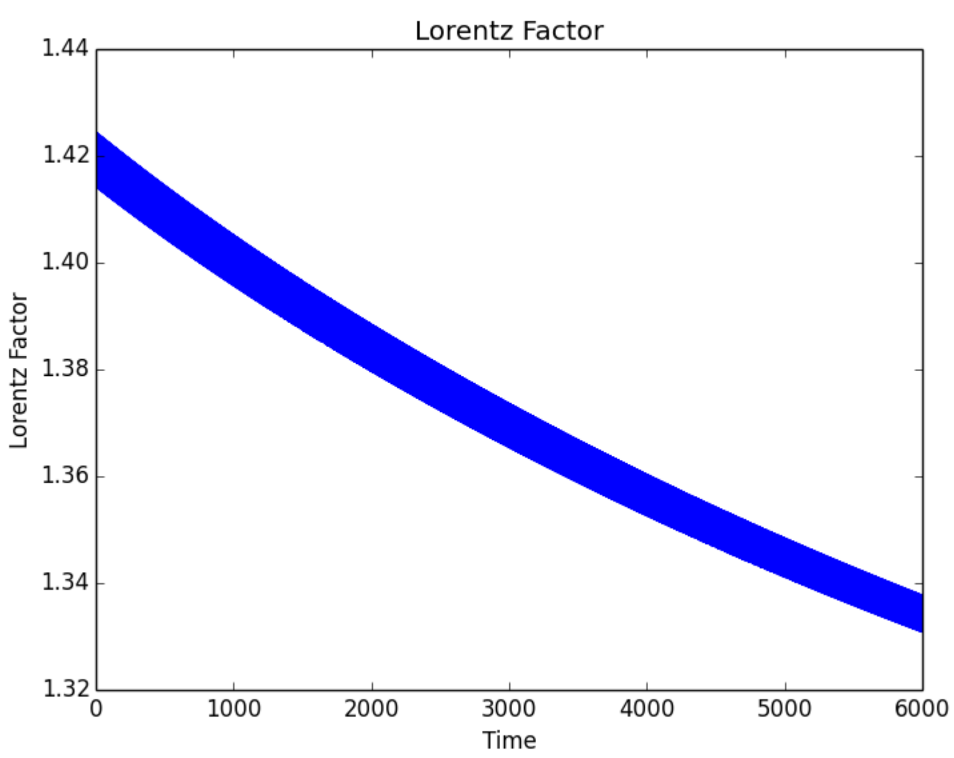
trajectory

1st & last of 50 large orbits

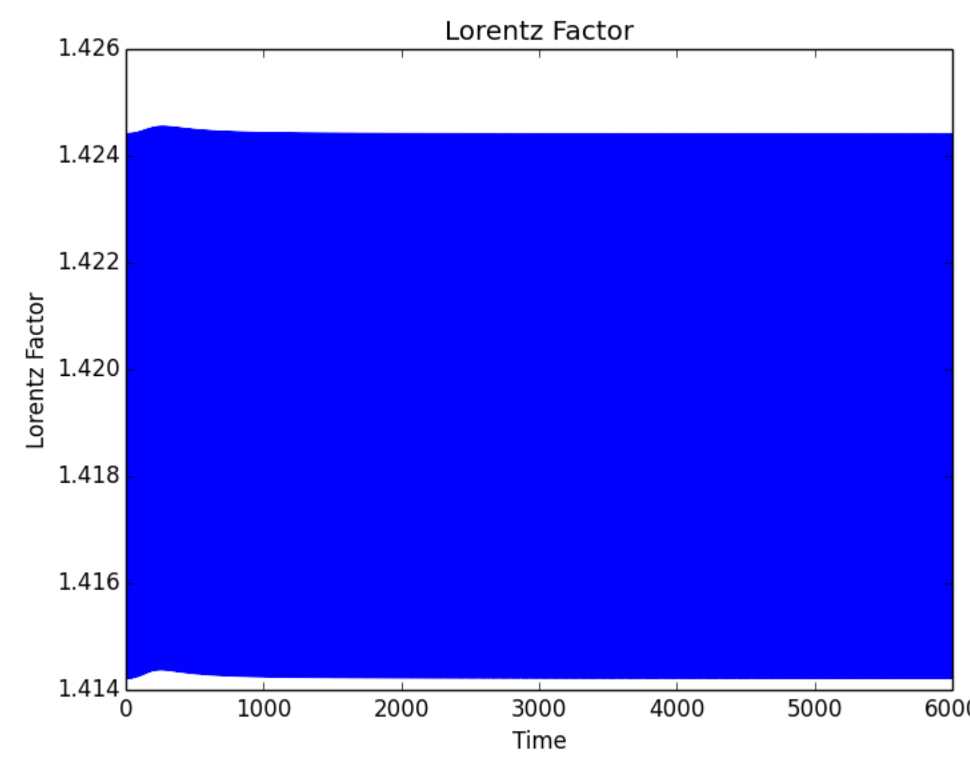


3rd order Runge-Kutta

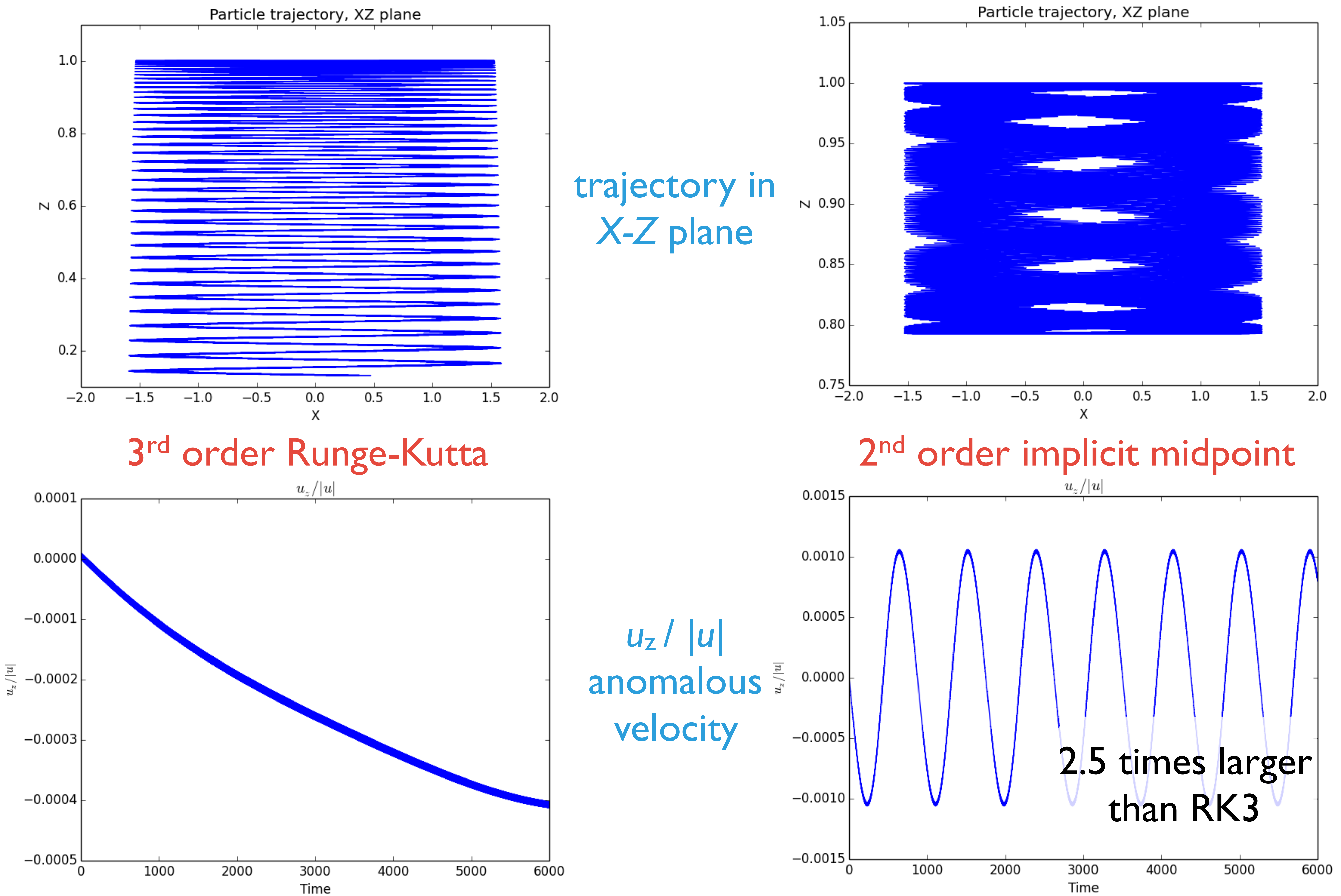
2nd order Störmer-Verlet



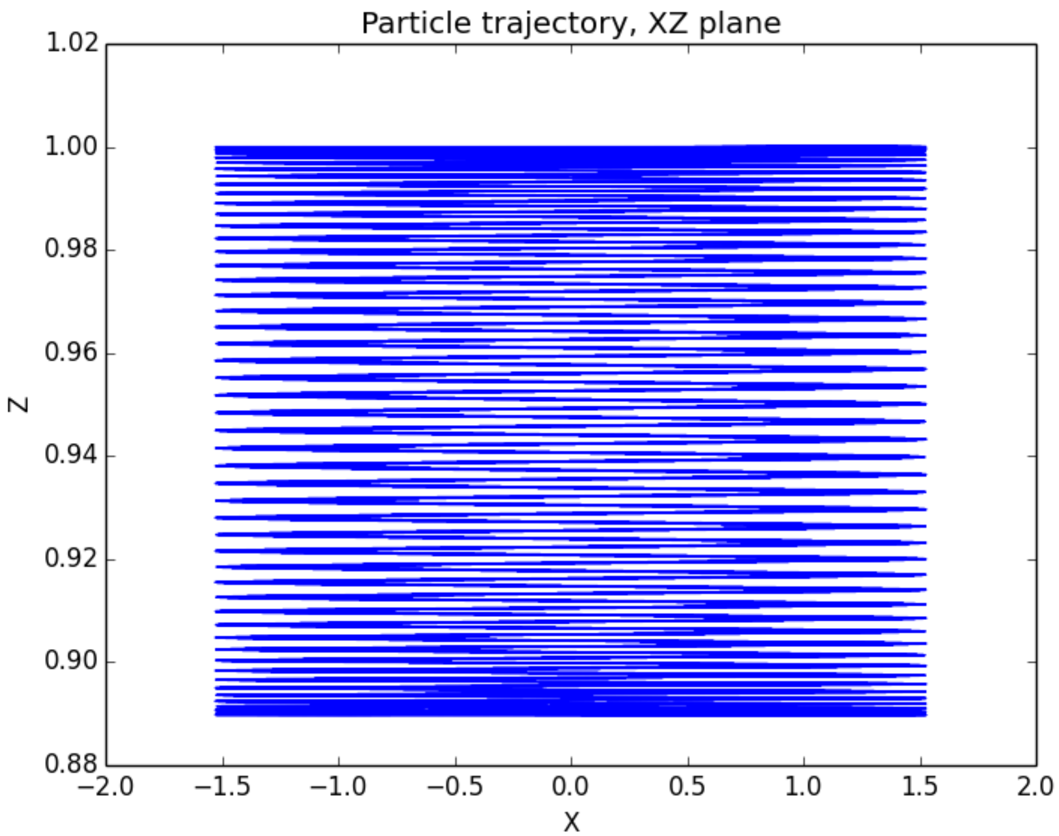
Lorentz factor



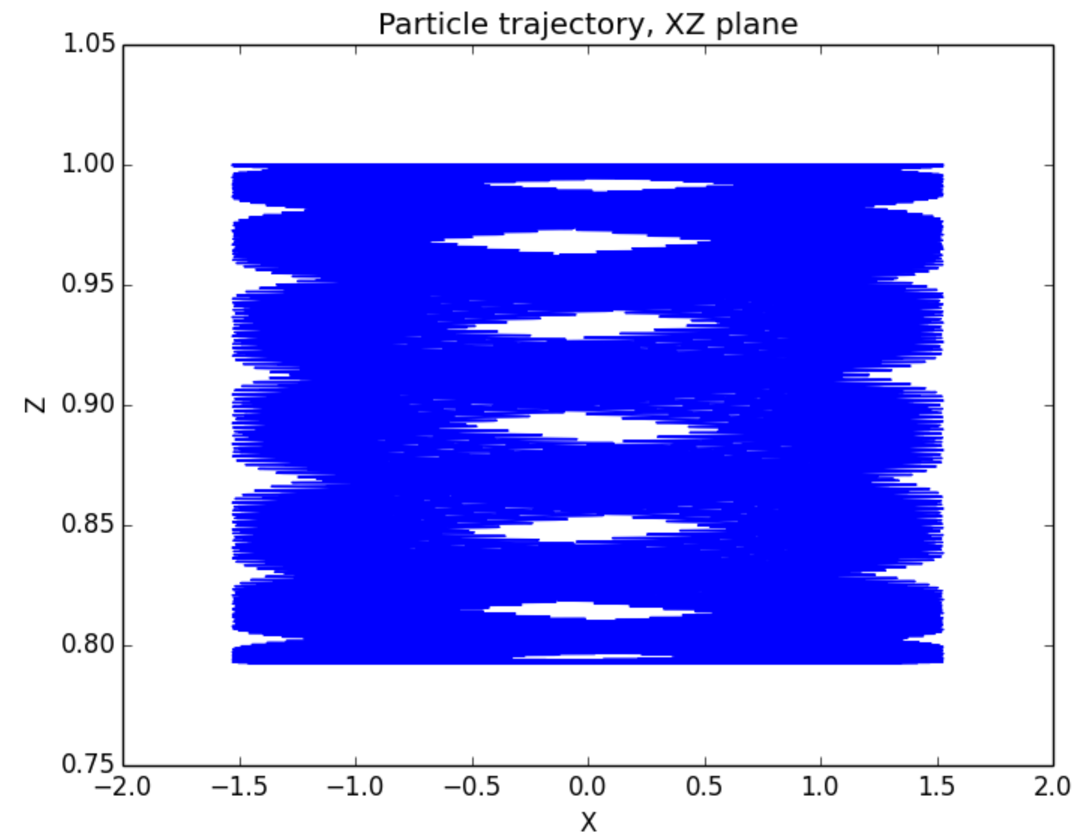
Momentum ⊥ to orbital plane



Momentum ⊥ to orbital plane

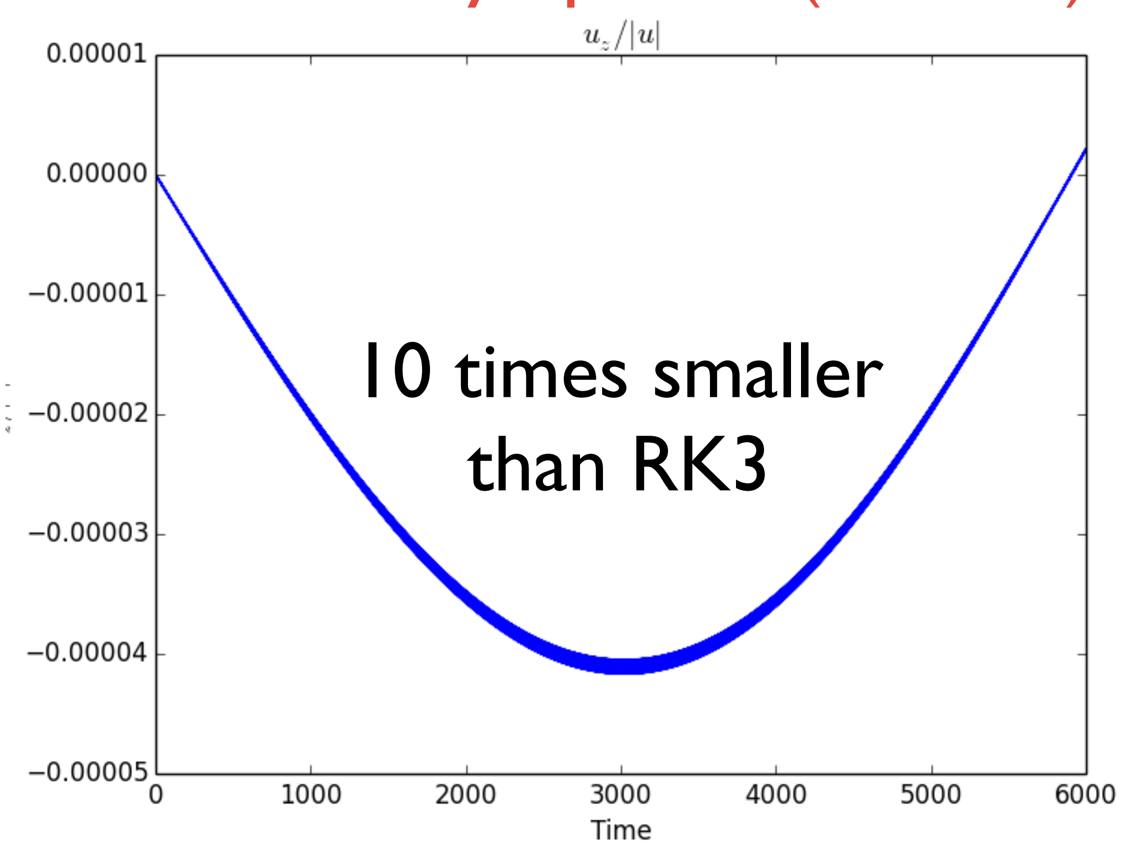


trajectory in *X*-*Z* plane

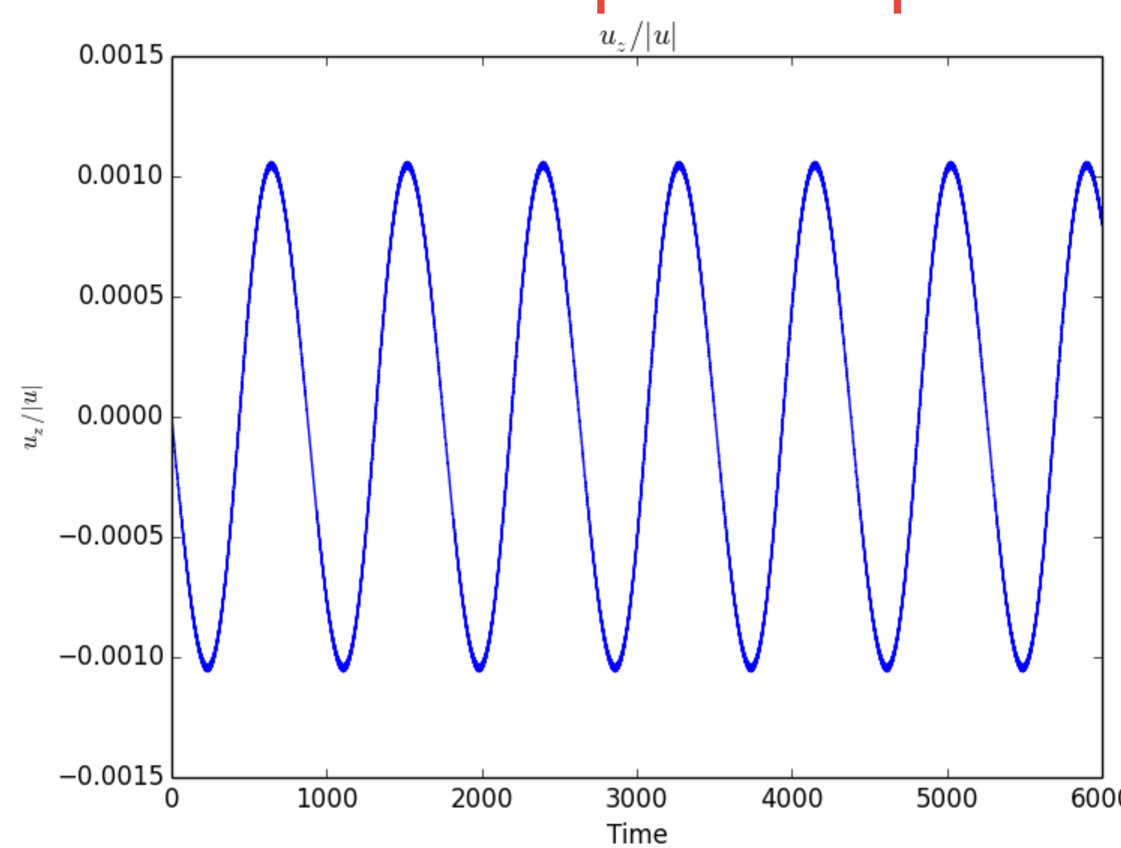


4rd order symplectic (Yoshida)

2nd order implicit midpoint



u_z / $|u|$ anomalous velocity



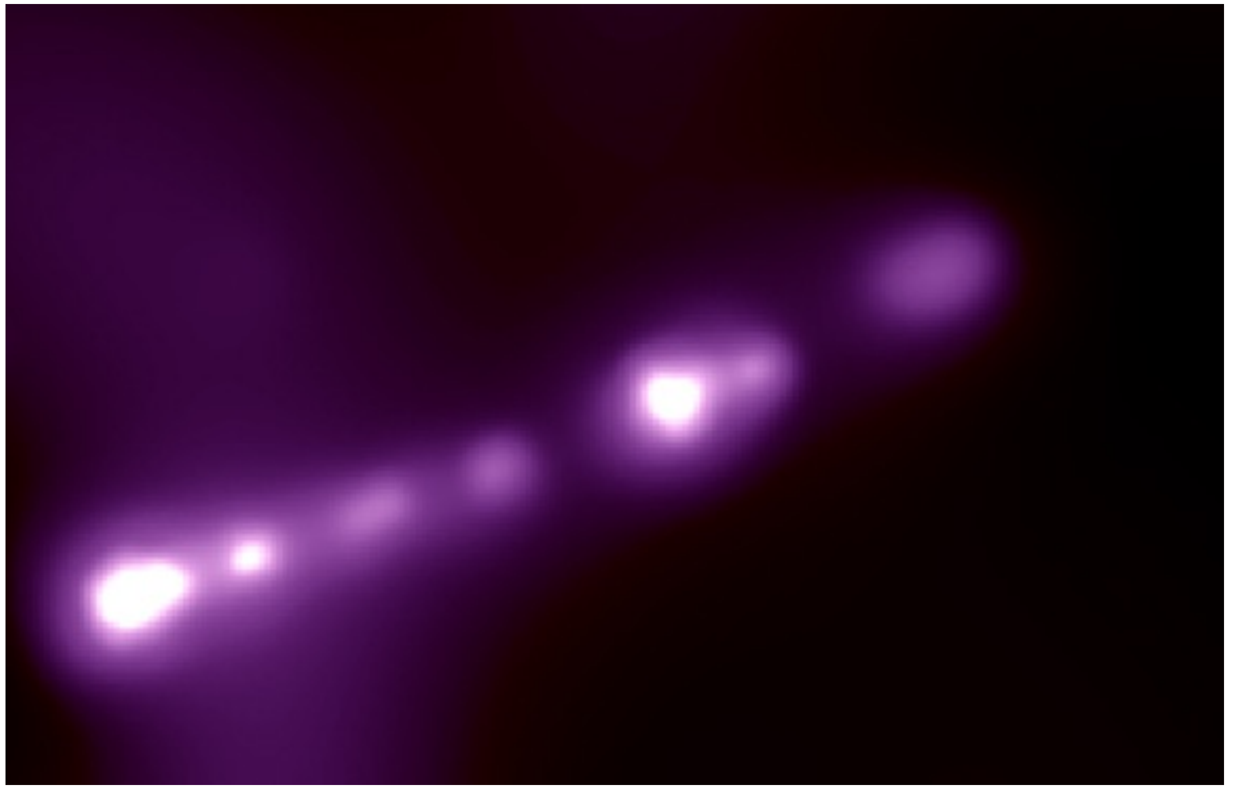
Summary

1. Many accretion flows are entirely collisionless, & parts of others
2. GR simulations in this regime haven't been performed
3. Accurate particle-in-cell simulations seem possible
4. Strang splitting of (a) Lorentz force in local orthonormal frame

   (b) symplectic solve for geodesic motion

   for particle integrator looks promising.

The stumble continues…



M87 jet

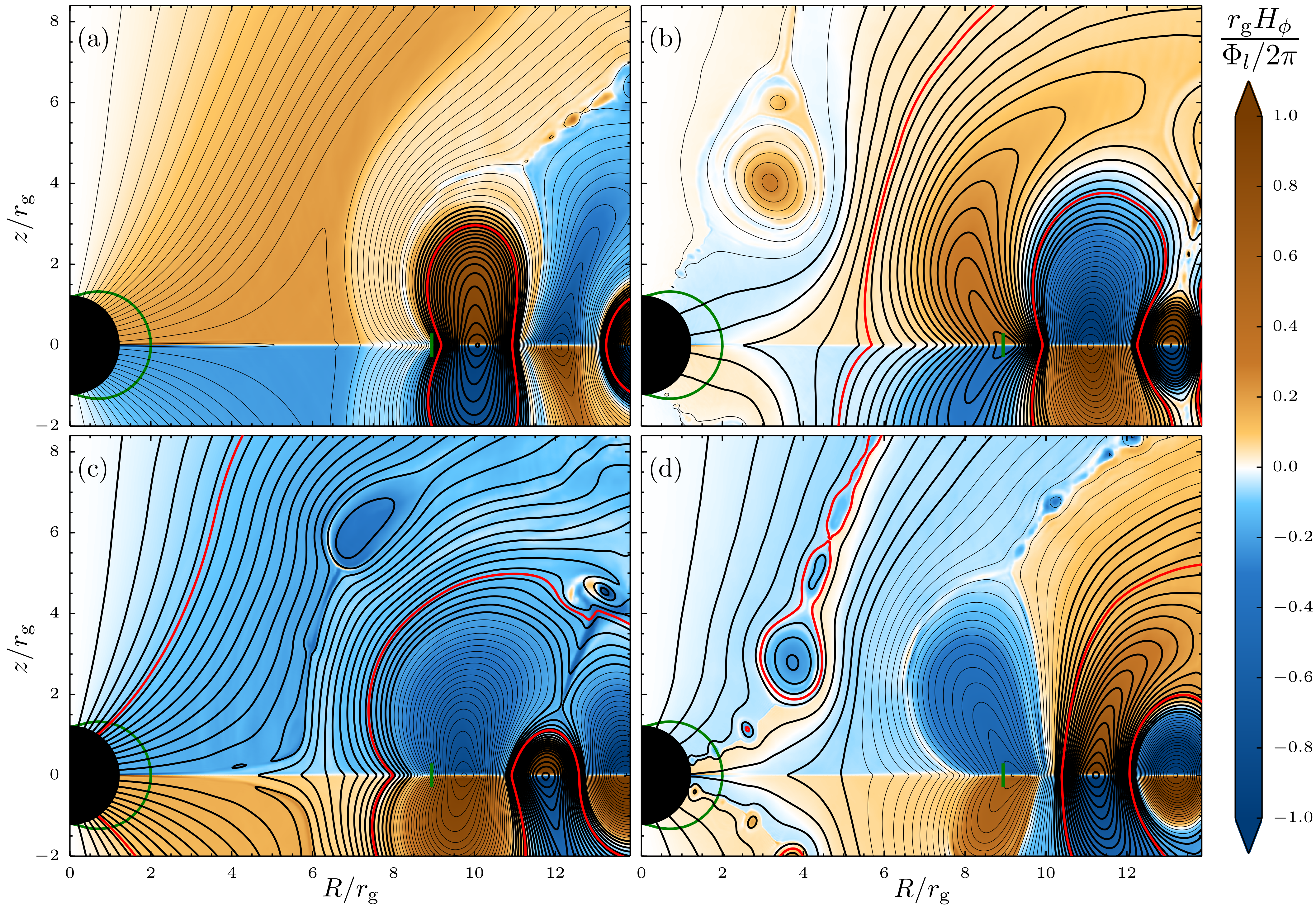
Chandra

EXTRA SLIDES

Black hole jet formation

PHAEDRA FFE

KP, Giannios, Beloborodov 2015

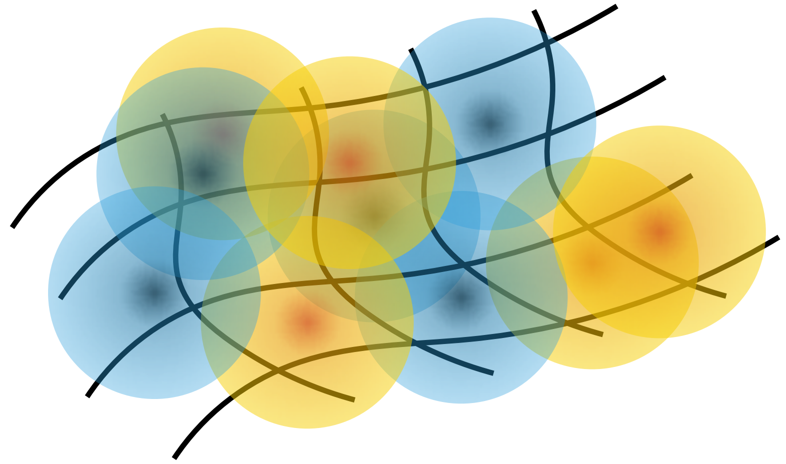


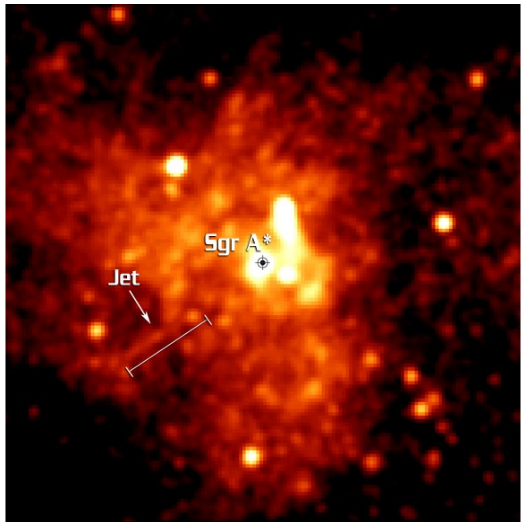
New jet model: powered by small-scale magnetic field systems

Prolific reconnection above disc → heats X-ray corona?

Curved spacetime plasma kinetics

Particle-in-cell method





galactic centre black hole

EM fields: multidomain staggered-Chebyshev spectral method

Kopriva 1994, 1996, 1998

Constraint equations: hyperbolic-parabolic generalised Lagrange multipliers

Munz+ 2000, Dedner+ 2002

PROGRESS

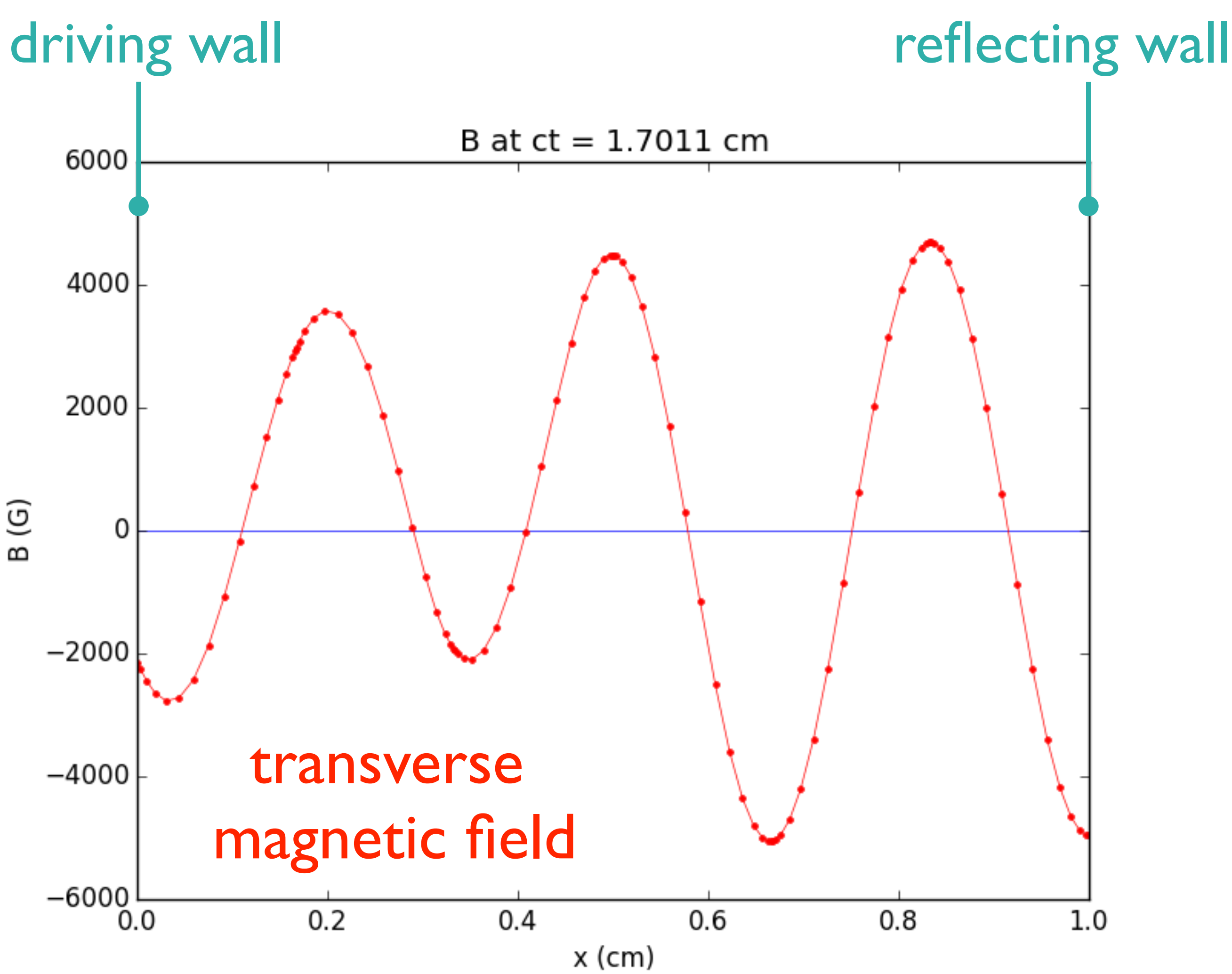
- 1D test code in flat spacetime
- 3+1 particle equations from Hamiltonian
- Phase-space-volume conserving particle integrator
  - Strang split:
    Boris move (EM force) + implicit symplectic (metric terms)

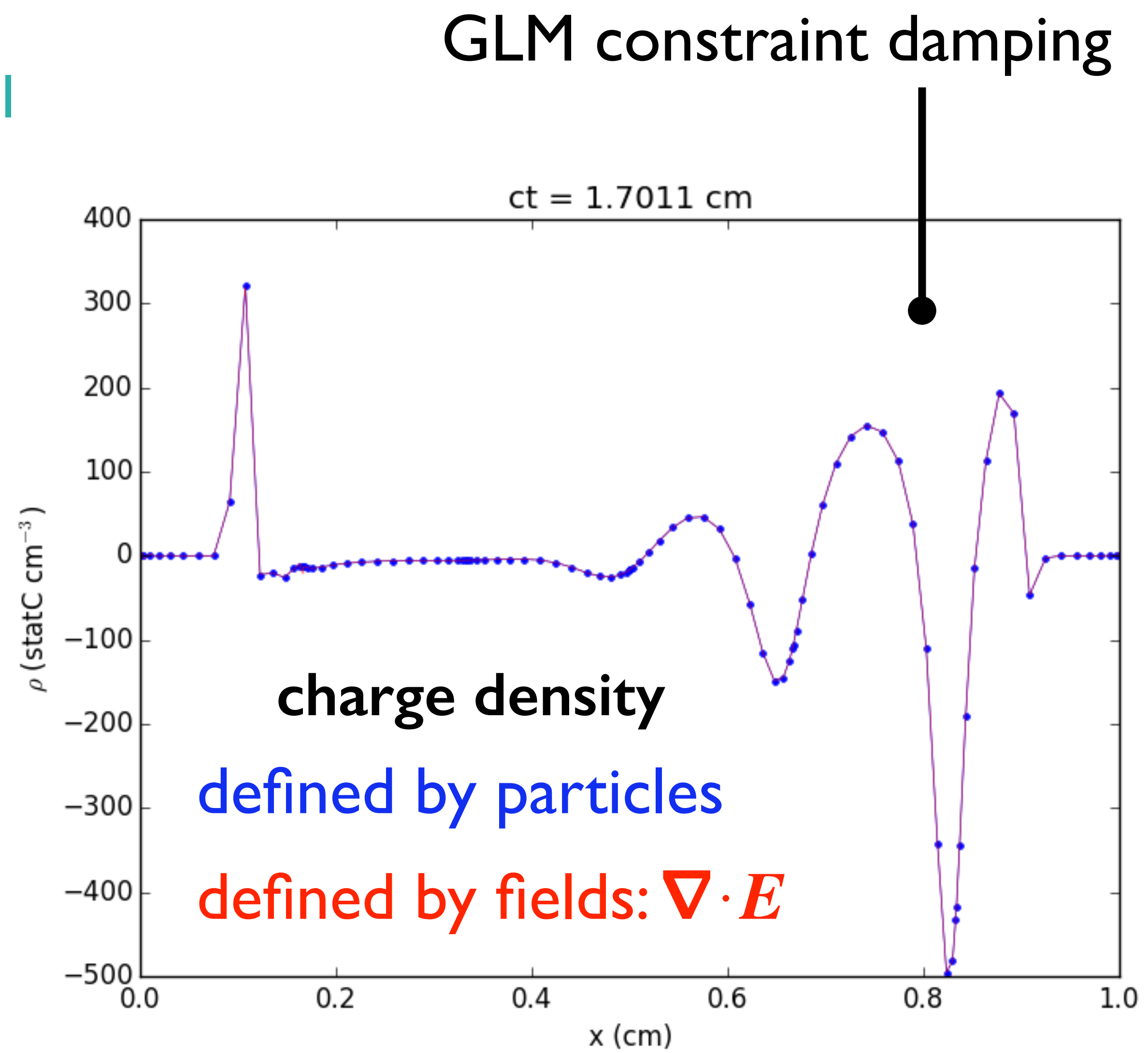
1D pathfinder code

Example: transverse EM wave driven through plasma

6 x 16-point elements

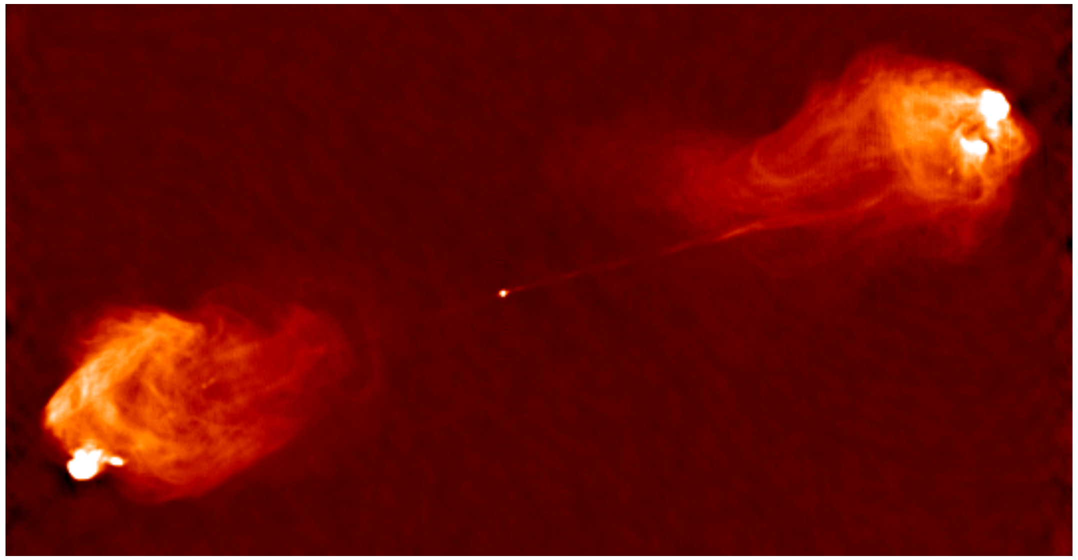
flat spacetime





Plasmas around black holes & neutron stars

VLA radio data



relativistic jets

artist's conception (NASA)

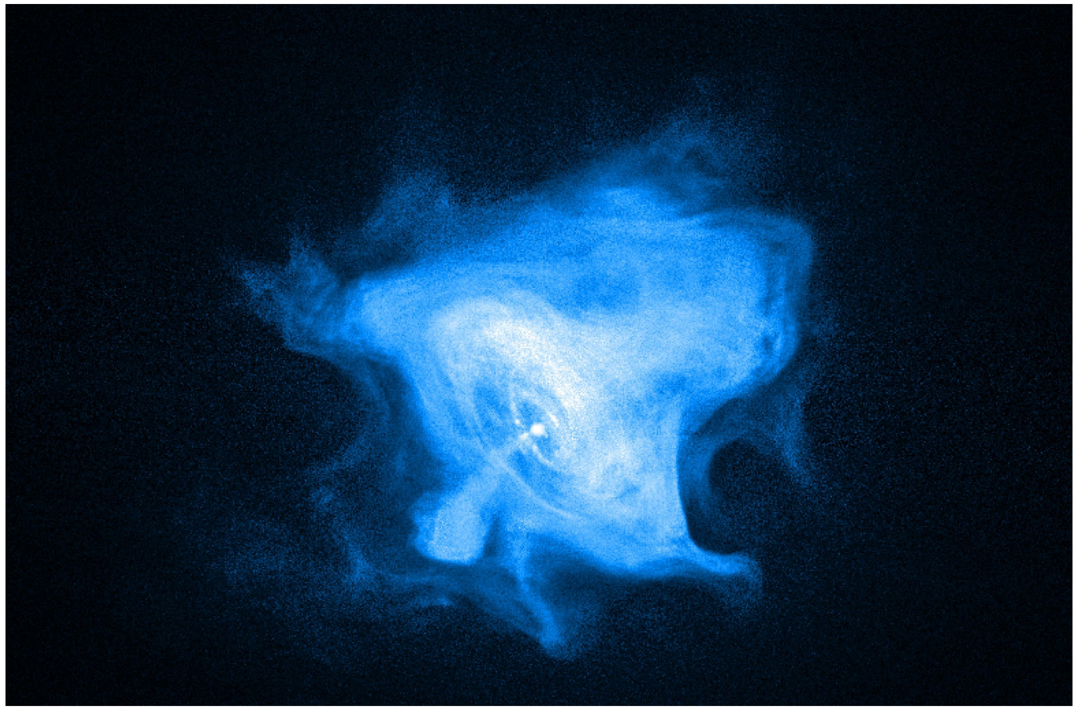
magnetars

nature's most extreme & exotic environments

accretion discs

artist's conception (*Interstellar*)

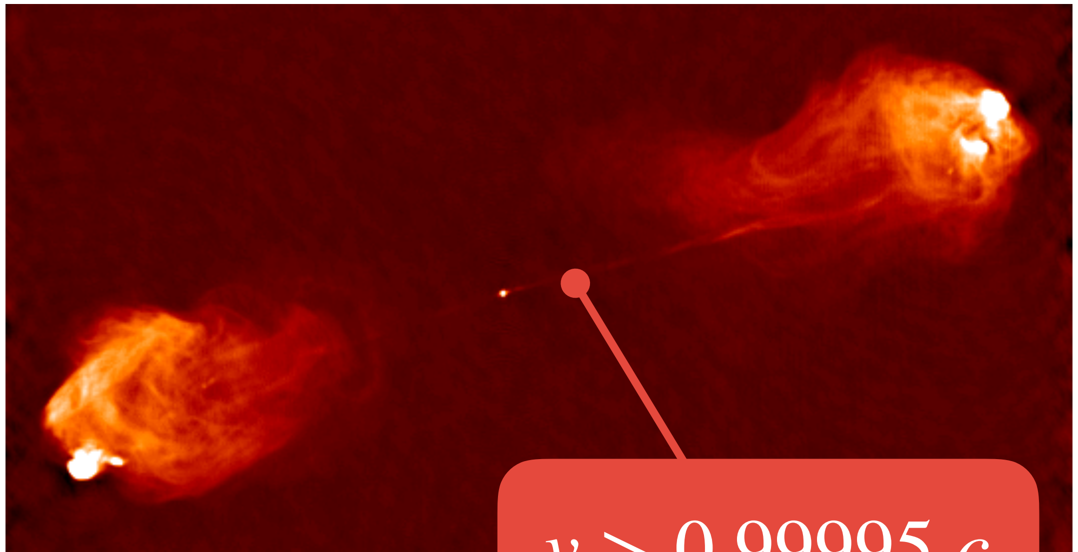
pulsars



Chandra X-ray data

Plasmas around black holes & neutron stars

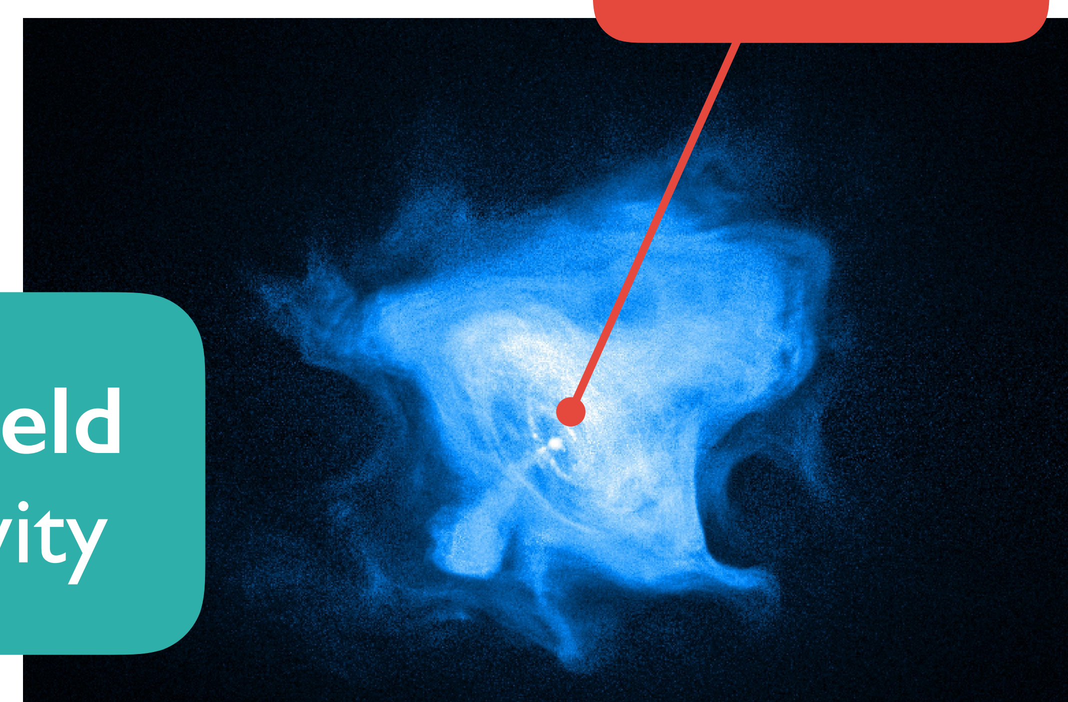
VLA radio data



artist's conception (NASA)

nature's most extreme & exotic environments

artist's conception (*Interstellar*)



Chandra X-ray data

all in strong-field general relativity