Memorandum

To: L3 Distribution

From: F. Primini

Subject: Computing L3 Limiting Sensitivity Maps

Date: 08/01/07

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## 1 Introduction

I define “limiting sensitivity” to be the flux of a point source that meets but does not exceed the flux significance threshold for inclusion in the L3 catalog. It is a function of source position, background, and the algorithm used to calculate flux and flux significance. It differs from and is higher than the source detection limit (e.g., a 5 count source in a low background observation can be detected at high confidence, but its flux significance is less than $3\sigma$ ).

Limiting Sensitivity is a “field property” (like the exposure map). It can be used to provide a simple x-ray flux limit for individual sources detected at other wavelengths, and is required to calculate sky coverage histograms (solid angle surveyed as a function of limiting flux), which are themselves needed to calculate luminosity functions and *logN-logS*. For the L3 catalog, a full-field sensitivity map will be provided for each observation, for each science energy band (RD v. 0.6, Sec. 2.1.5).

In an earlier e-mail (7/6/07) I provided a draft specification. I review that here, for the record, and provide an updated version.

## 2 Initial Specification

The initial specification used the L3 model background map directly to estimate the background counts $B$ in a circular source aperture. The minimum total counts $n_{min}$ that would yield a flux at the significance threshhold $SNR_{min}$ is given by

$$\frac{n_{min} - B}{\sqrt{n_{min} + var(B)}} = SNR_{min} \quad , \tag{1}$$

or

$$n_{min} = B + \frac{SNR_{min}^2}{2} \left\{ 1 + \sqrt{1 + \frac{4(B + var(B))}{SNR_{min}^2}} \right\} \quad . \tag{2}$$

The limiting sensitivity $S_{min}$ is then (in photon flux)

$$S_{min} = \frac{n_{min} - B}{\alpha\, E_s} = \frac{SNR_{min}^2}{2} \left\{ 1 + \sqrt{1 + \frac{4\,(B + var(B))}{SNR_{min}^2}} \right\} (\alpha\, E_s)^{-1} \quad , \tag{3}$$

where α and $E_s$ are the encircled counts fraction and average exposure map value in the source aperture. The variance of the background was estimated by computing the rms of individual background map values near the source aperture.

$S_{min}$ is calculated for each image pixel in the background map, using a circular, 90% ECF source aperture appropriate for that image location. For each energy band, only a single resolution sensitivity map is generated, from the smallest blocking factor background map that includes the entire field-of-view.

## 3 Updated Specification

The limiting sensitivity as described above is inconsistent with the way that we compute actual source fluxes. We do not estimate background in source apertures from model background map data in the aperture, but rather from image data in an annular aperture surrounding the source aperture, which may include some source contribution. For future reference, I'll call this the “local background” approach. Moreover, the background variance is that appropriate to raw counts in the local background aperture, not modeled background counts. These inconsistencies are addressed in the updated specification.

### 3.1 Randomized Background Map

The first step is to generate a background map consistent with the statistics of the actual image. Assuming that the model map accurately represents the mean background in the image, this may be accomplished by sampling Poisson-distributed random numbers, using the model map values as the means, i.e., for each image pixel in the input model map, generate a random number from a Poisson distribution with that pixel's model map value as mean. The resulting randomized background map (rbkmap) is used in all subsequent limiting sensitivity calculations. An example map is shown in Figure 3.1.

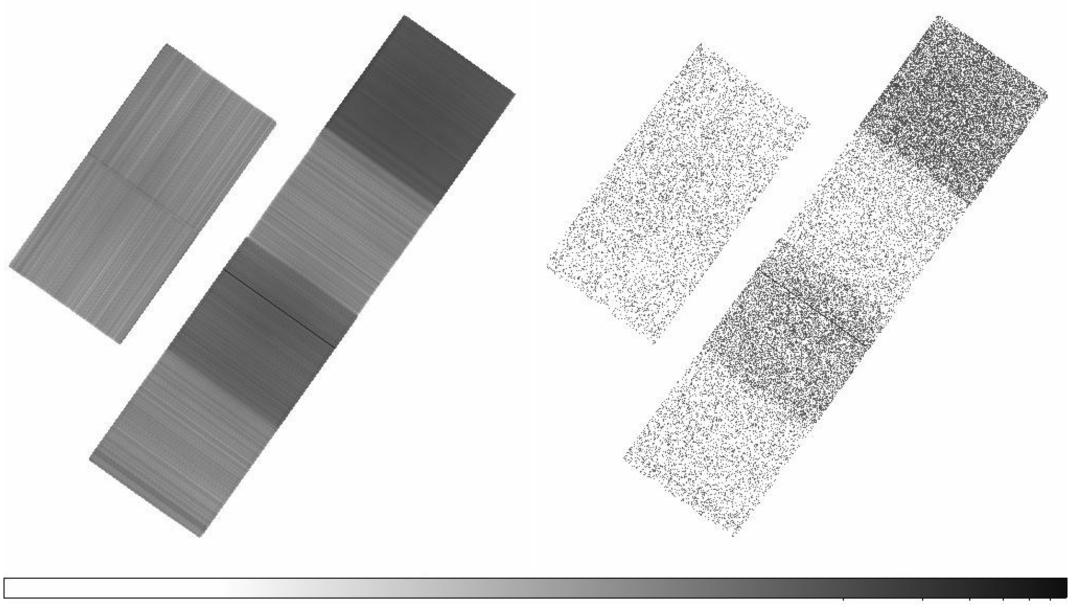


*Figure 3.1.: Model and randomized background maps for OBSID 786, broad band, blocking factor = 4.*

## 3.2 Limiting Sensitivity Algebra in the Local Background Case

The limiting sensitivity formula for the local background case is slightly different from the one I circulated earlier. The details are worked out here, using the same basic formalism described in the June 6, 2007 specification for computing aperture photometry quantities. For example, for photon flux, the relevant quantities are

$$
\begin{aligned}
n &= \text{Counts in Source Aperture} \\
m &= \text{Counts in Background Aperture} \\
\alpha &= \text{ECF in Source Aperture} \\
\beta &= \text{ECF in Background Aperture} \\
E_S &= \text{Average Source Aperture Exposure Map Value} \\
E_B &= \text{Average Background Aperture Exposure Map Value} \\
f &= \alpha E_S \\
g &= \beta E_B \\
A_S &= \text{Source Aperture Area} \\
A_B &= \text{Background Aperture Area} \\
r &= A_B E_B / A_S E_S
\end{aligned}
\tag{4}
$$

and the net photon flux and error in the classical limit are given by

$$
\begin{aligned}
S &= \frac{rn-m}{rf-g} \\
\sigma_S &= \frac{\sqrt{r^2 n+m}}{rf-g}
\end{aligned}
\quad . \tag{5}
$$

The limiting sensitivity is again determined by finding the value of $n_{min}$ such that

$$
\frac{rn_{min}-m}{\sqrt{r^2 n_{min}+m}} = SNR_{min} \quad . \tag{6}
$$

The solution here is

$$
rn_{min} = m + \frac{r\, SNR_{min}^2}{2}\left\{1+\sqrt{1+\frac{4\,m}{r\, SNR_{min}^2}\left(1+\frac{1}{r}\right)}\right\} \quad , \tag{7}
$$

and the limiting sensitivity is then determined from Eq. 5:

$$S_{min} = \frac{rn_{min} - m}{rf - g} = \frac{r\, SNR^2_{min}}{2} \left\{ 1 + \sqrt{1 + \frac{4\, m}{r\, SNR^2_{min}} \left( 1 + \frac{1}{r} \right)} \right\} (rf - g)^{-1} \quad . \tag{8}$$

Similar relations for limiting sensitivity in net counts, rate, or energy flux apply, with the substitution of the appropriate *r, f,* and *g* for those quantities.

### 3.3 Summary of Steps in Updated Specification

The steps in computing the limiting sensitivity map may then be summarized as follows:

For each energy band:

1. Select the model background map at the smallest blocking factor that includes the entire FOV (typically bl=4 for ACIS) and the corresponding exposure map;
2. Generate the randomized background map (rbkmap) as discussed in Section 3.1 above;
3. Create an output limiting sensitivity map with the same blocking factor, dimensions, and WCS;
4. For each image pixel in the rbkmap that falls within the FOV:
    a. Determine a circular source aperture corresponding to the 90% (parameter) ECF at that image location (e.g., from mkpsfsize);
    b. Determine a circular annular background aperture surrounding the source aperture; the annulus inner radius should be the radius of the source aperture; the annulus outer radius should be 5 (parameter) times the inner radius;
    c. Determine the areas $A_s$ and $A_B$ of the apertures; the background aperture area should include only that part that is within the FOV;
    d. Determine the average exposure map values $E_S$ and $E_B$ in the apertures; again, only the part of the background aperture within the FOV should be used;
    e. Determine the number of counts *m* in the background aperture;
    f. Determine the ECF $\beta$ in that part of the background aperture contained in the FOV; the ECF $\alpha$ in the source aperture is assumed to be 0.9;
    g. Compute the limiting sensitivity for this image pixel in the output map, using Eq. 8 and the definitions for *r*, *g*, and *f* in Eq. 4; use $SNR_{min}$ = 3 (parameter).

## 4 Approximations

A useful approximation may significantly improve the run-time of what is likely to be an expensive task. In Eq. 8, we may write

$$(rf - g)^{-1} = \frac{1}{(rf - g)} = \frac{1}{rf\,(1 - \frac{g}{rf})} \approx \frac{1}{rf} \left\{ 1 + \frac{g}{rf} \right\} \quad , \tag{9}$$

since $g/rf \ll 1$ . We can, in fact, estimate $g/rf$ , assuming the background aperture size in Step 4b above, and the definitions for $r, g, f$ in Eq. 4:

$$\frac{g}{rf} = \frac{\beta E_B}{\frac{A_B E_B}{A_S E_S} \alpha E_S} = \frac{\beta}{\frac{A_B}{A_S} \alpha} < \frac{0.1}{0.9 \times 24} \approx 0.005 \quad . \tag{10}$$

Therefore, we can ignore $g/rf$, and even skip calculation of the background ECF (Step 4f), without making an error of more than ~0.5% in the limiting sensitivity. In this case, we can re-write Eq. 8 as

$$S_{min} = = \frac{SNR_{min}^2}{2f} \left\{ 1 + \sqrt{1 + \frac{4\,m}{r\,SNR_{min}^2} \left( 1 + \frac{1}{r} \right)} \right\} \quad . \tag{11}$$

A further approximation can be made if $E_S \approx E_B$. In this case, $r \approx A_B / A_S$, and the sensitivity map can be computed in units of net counts, with the conversion to photon flux taking place in a dmimgcalc step by dividing the "net counts" map by the exposure map (times 0.9). An example net counts map for OBSID 786 is shown in Fig. 4.1.

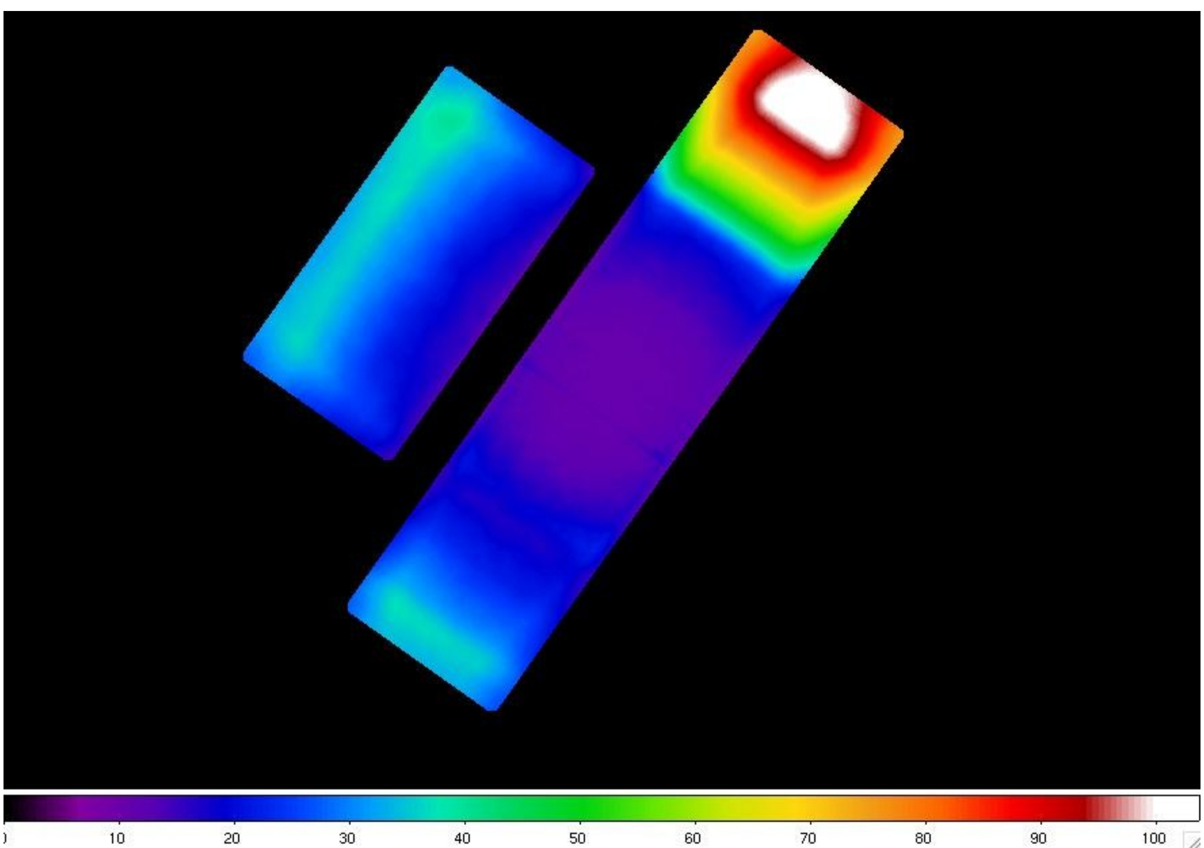


*Figure 4.1: Net Counts Limiting Sensitivity Map for OBSID 786, broad band, blocking factor = 4*