Cosmological weak lensing in full general relativity

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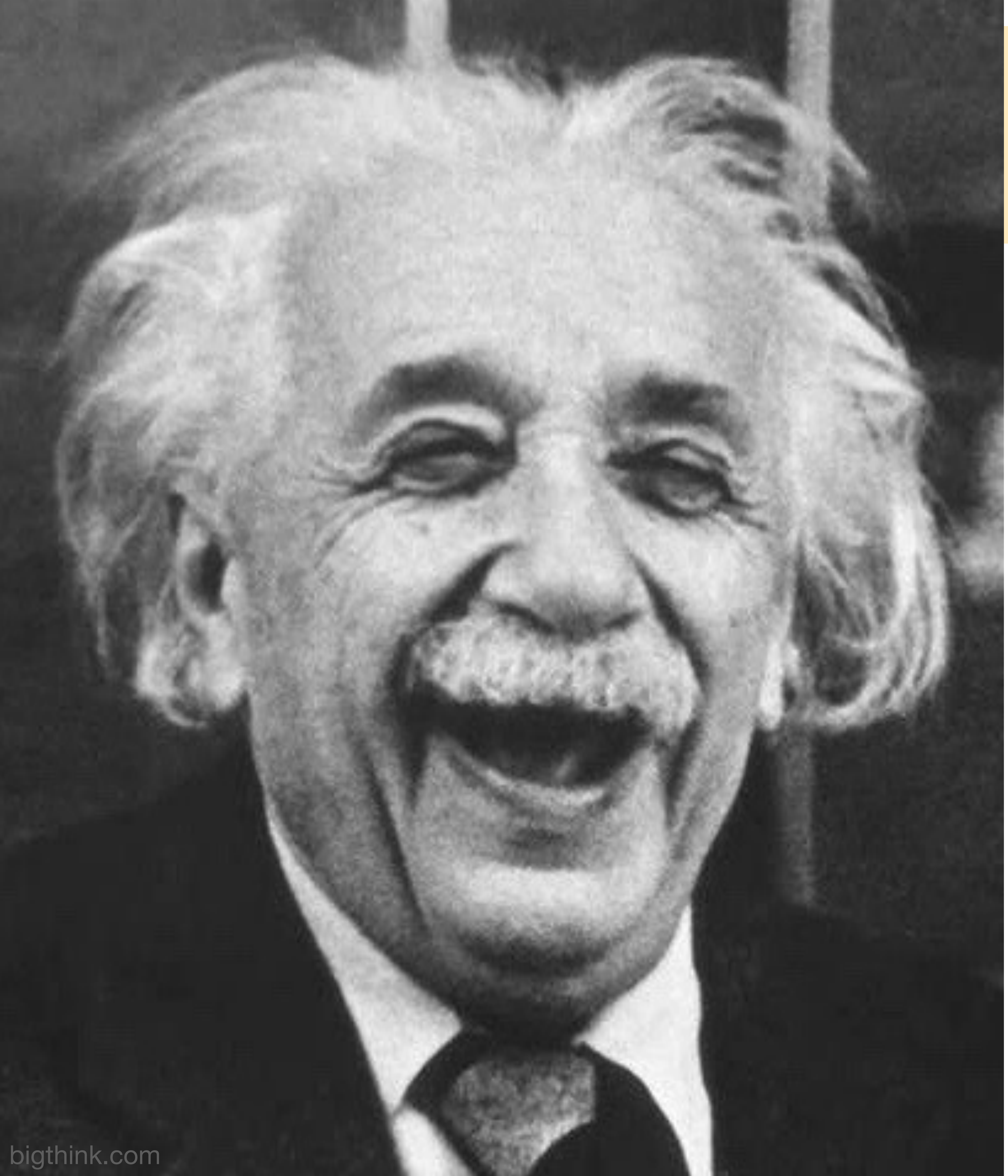
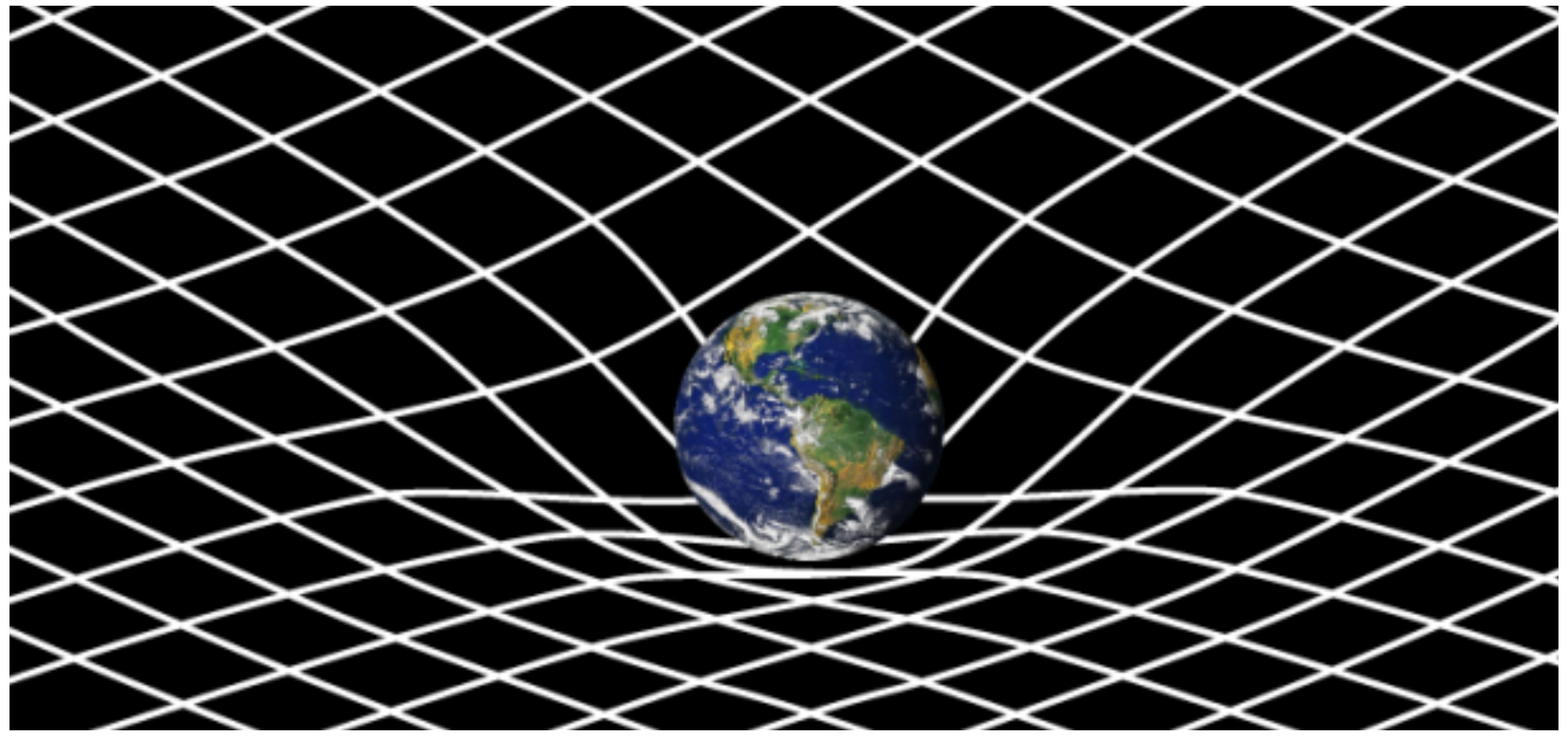
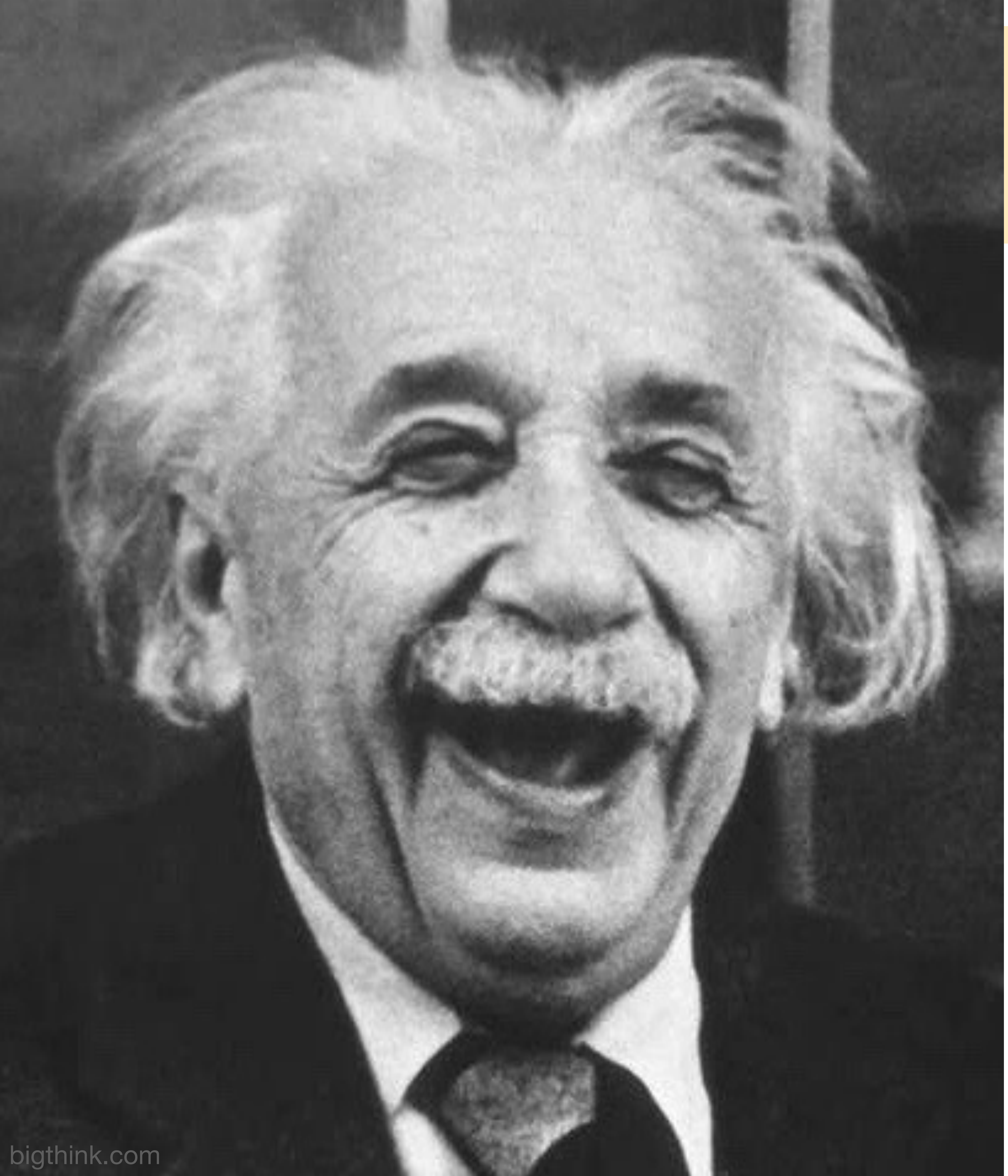


Modern cosmology is based on general relativity

This means that gravity is described by the geometry of *spacetime*



NASA



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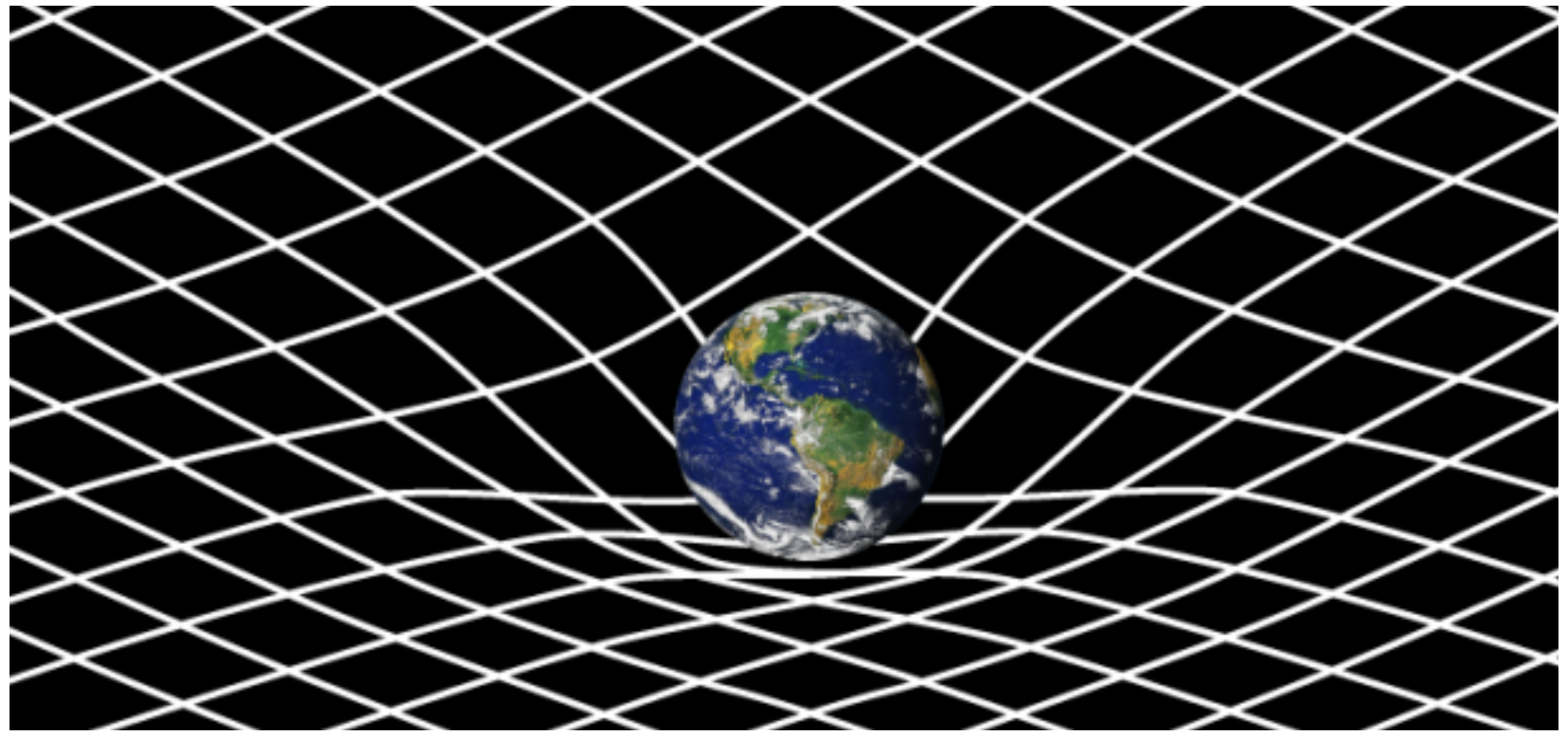

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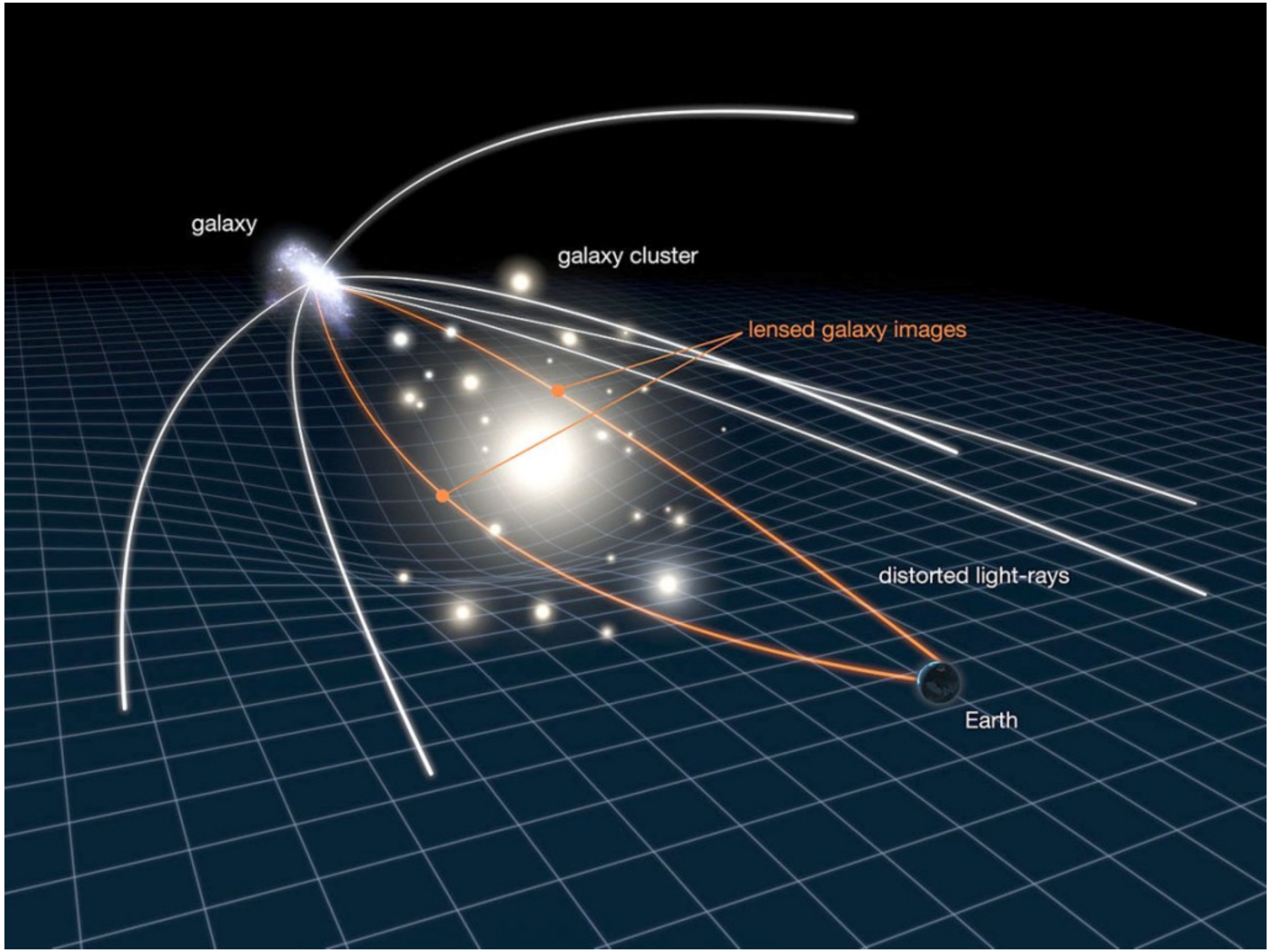
Gravitational lensing



Credit: NASA, ESA & L. Calçada

Galaxy images are distorted due to massive objects in the foreground

This is known as gravitational lensing

We measure this effect across the sky, which is related to the distribution of matter in the Universe

We can use this to constrain the cosmological model

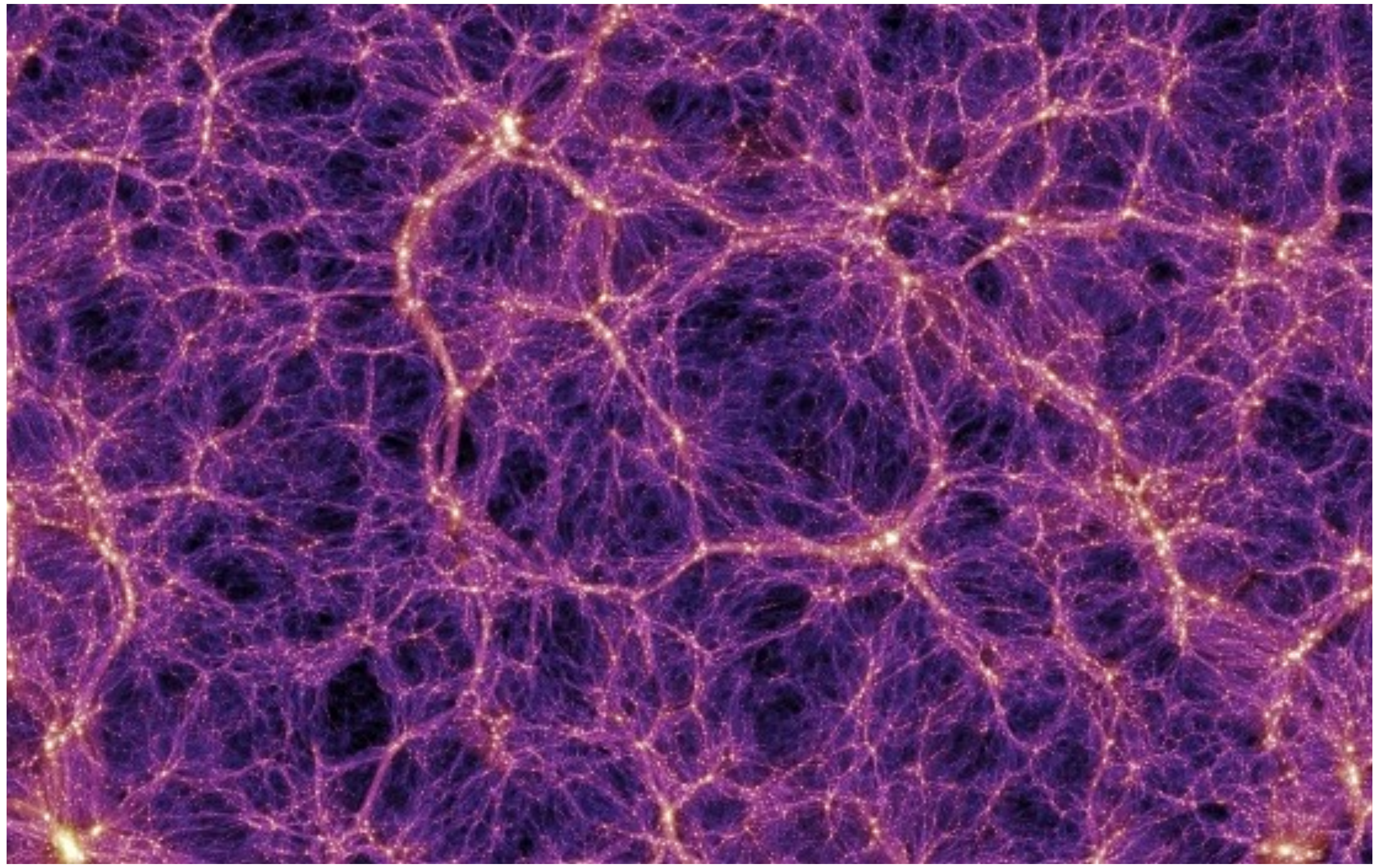
How do we do this?

We need to know the predicted signature for different theoretical models.

This is where simulations come in

Using N-body simulations, we can run a variety of different cosmological models

Then we can compare each lensing signal with our observations to see which one fits best

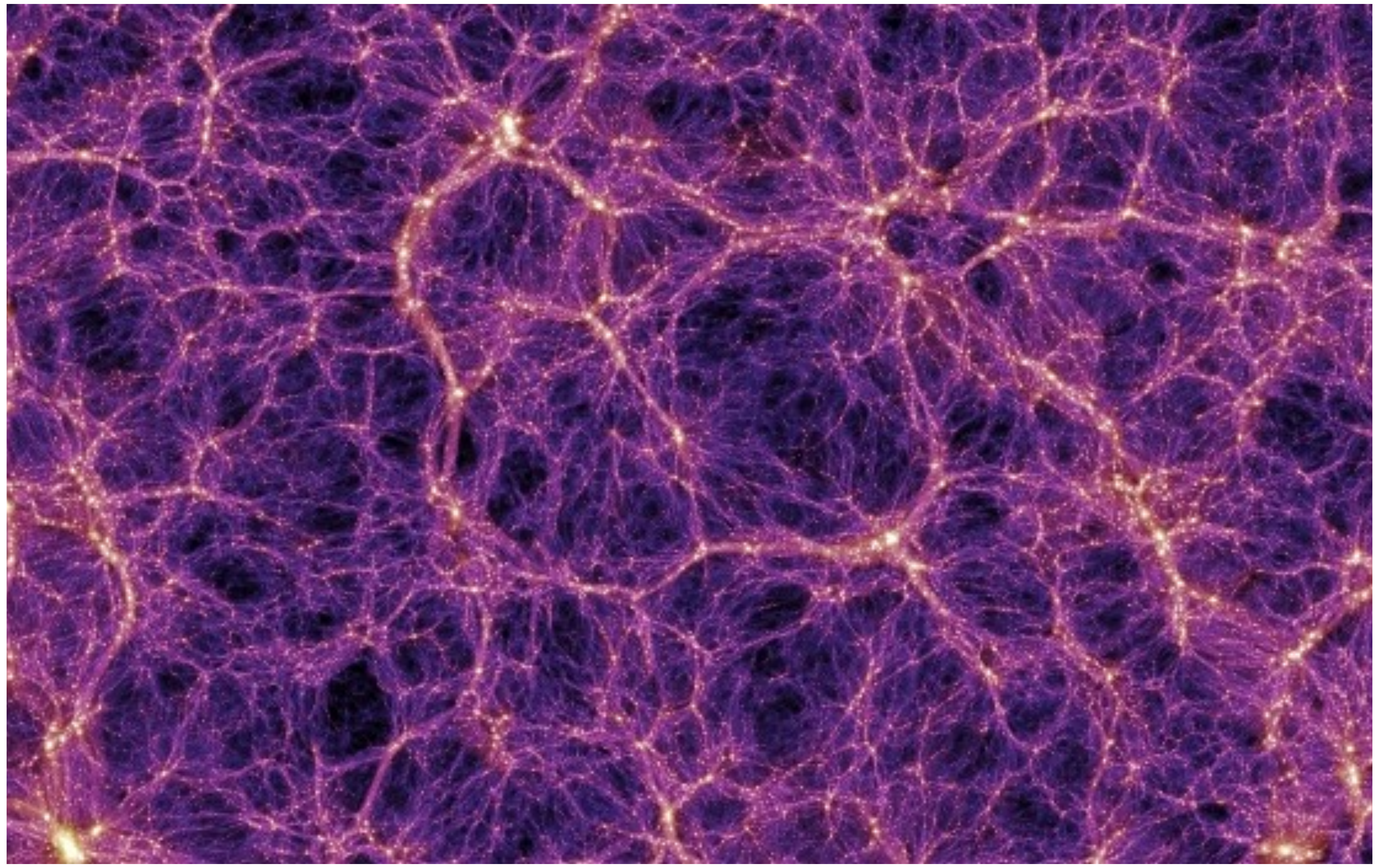


Springel et. al (2005)

How do we do this?

But …

Typically, these are Newtonian N-body simulations, which means there is no interaction between matter and space-time



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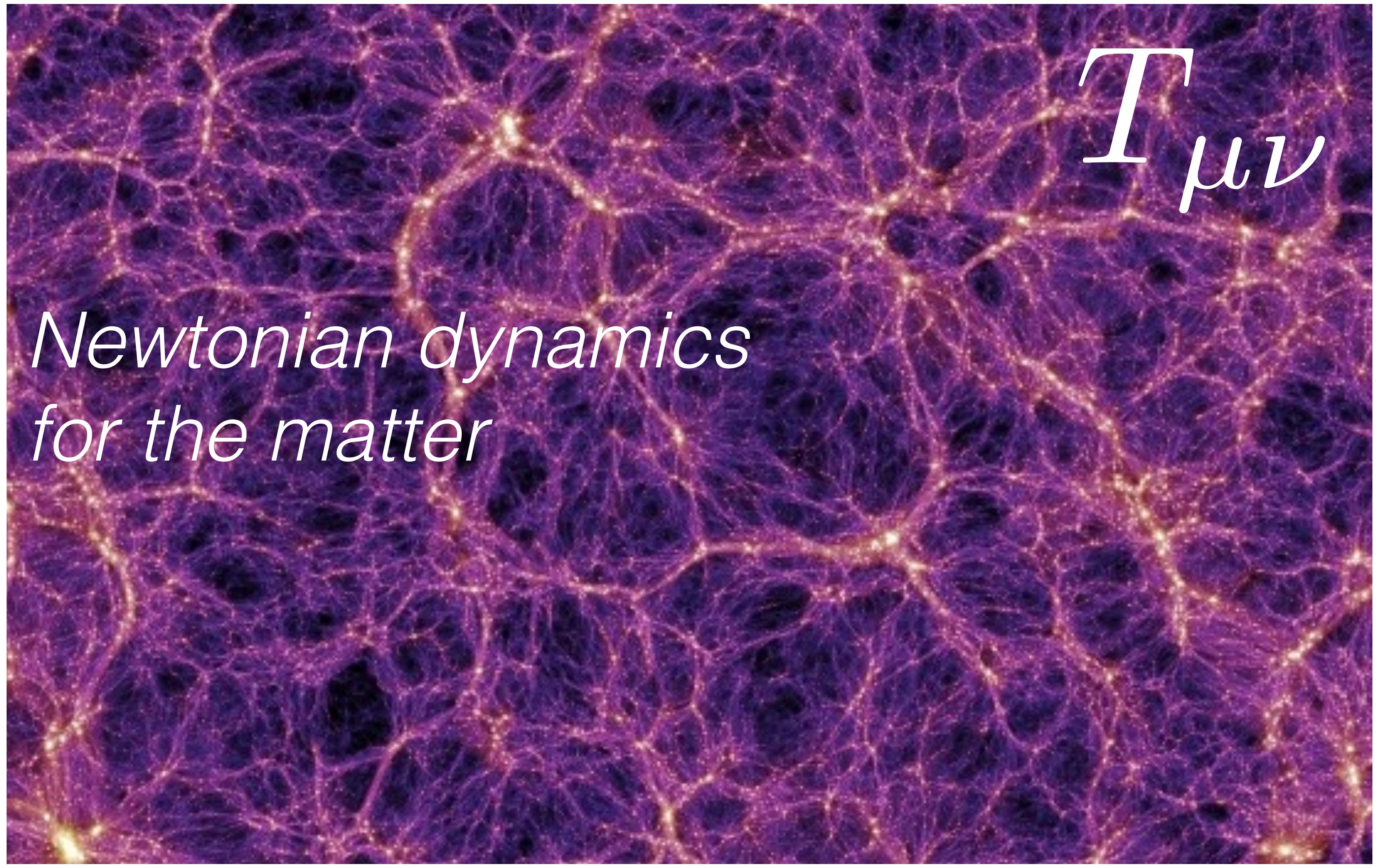

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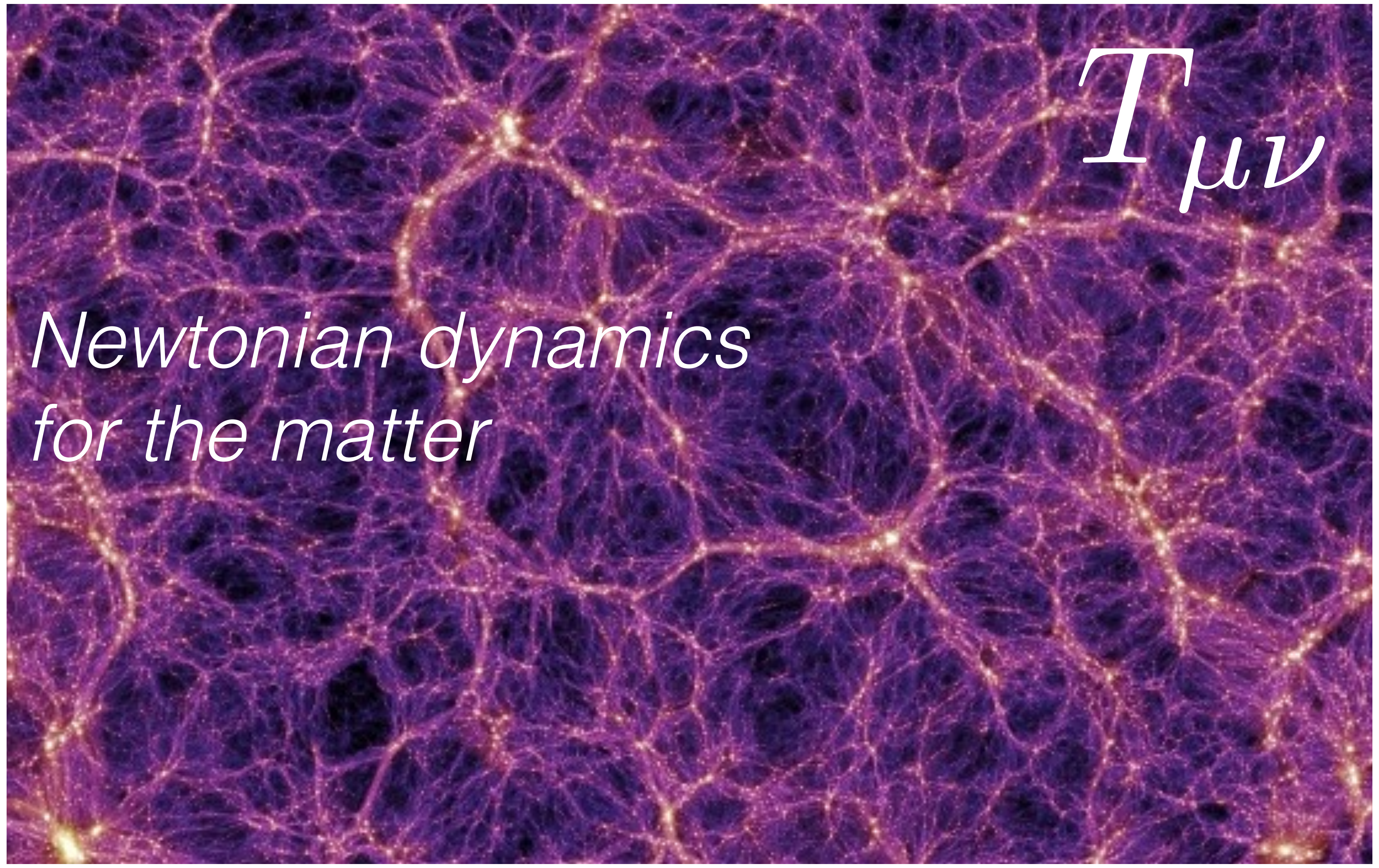

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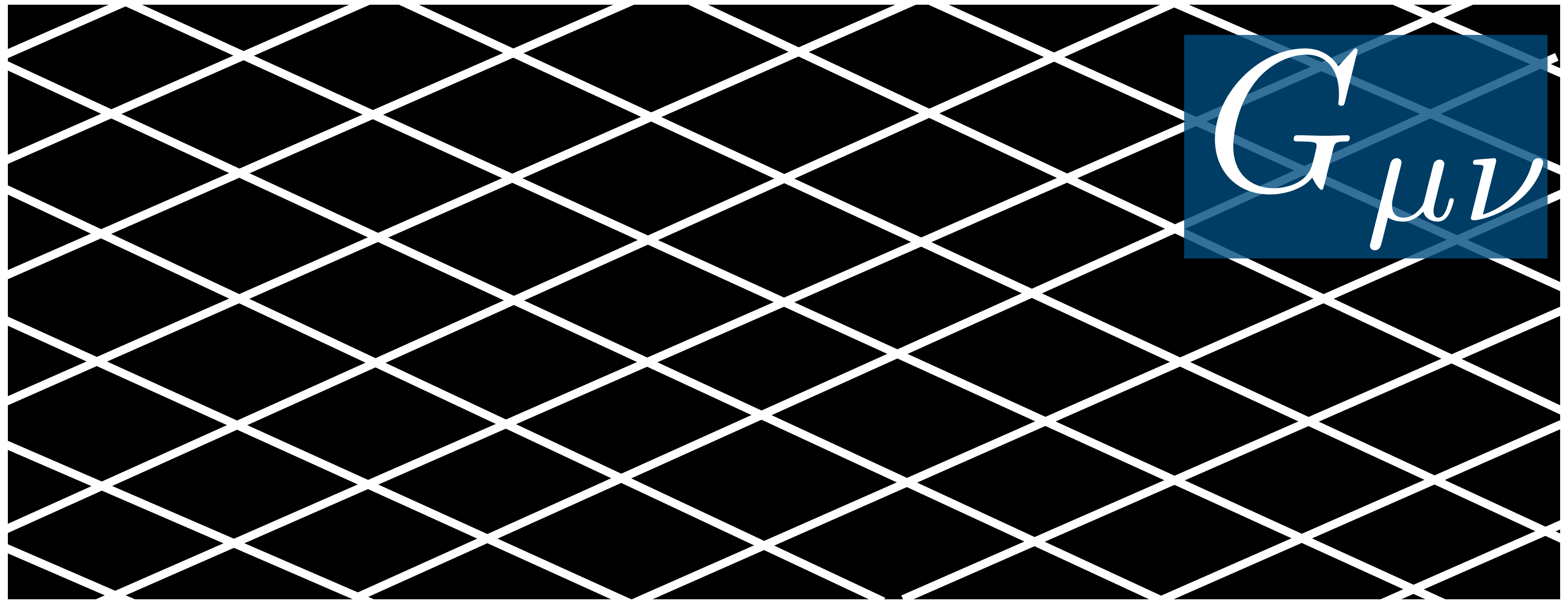

And GR for the homogeneous space-time

How do we do this?

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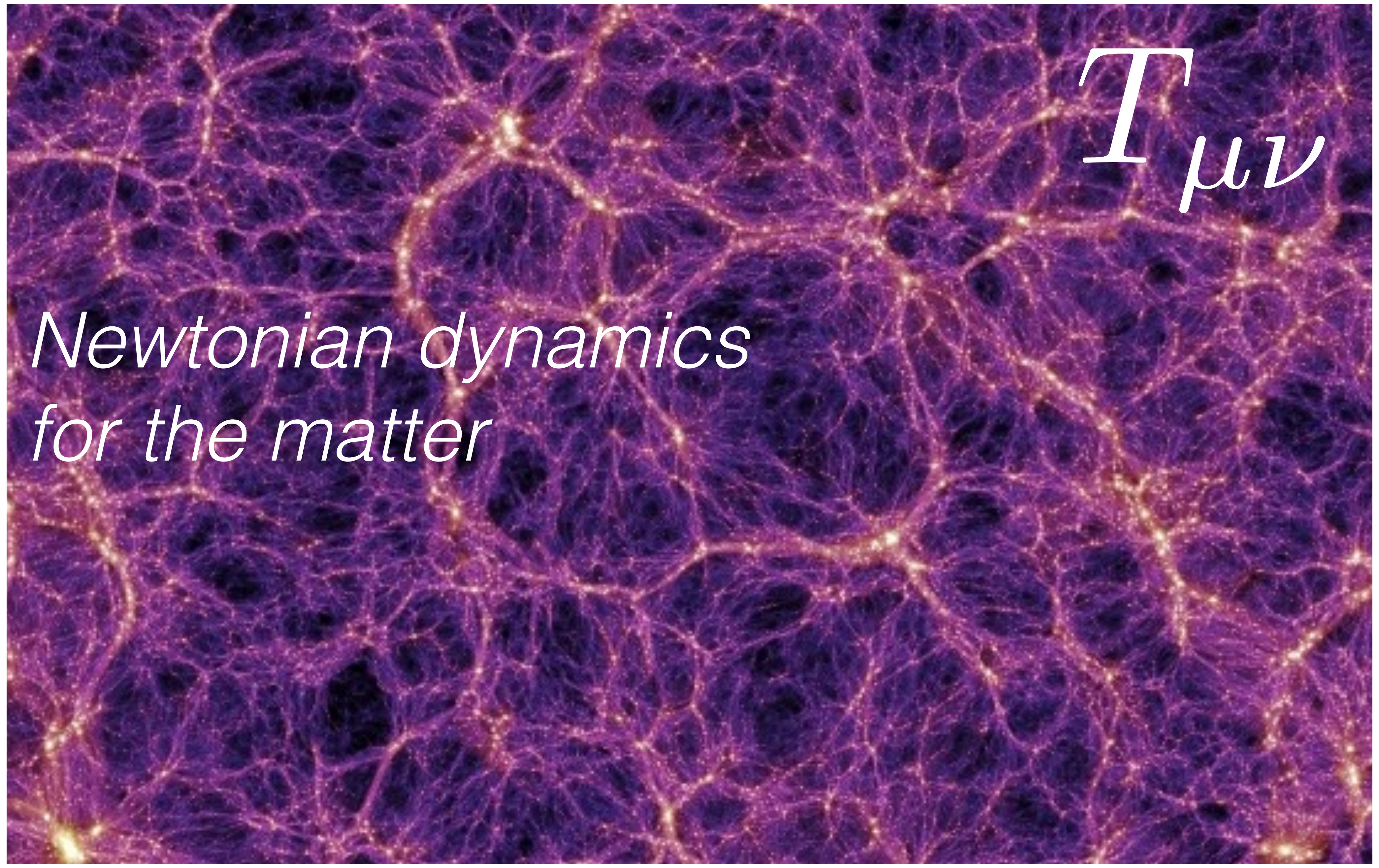
The Newtonian gravitational potential is used to predict the general-relativistic lensing signal

This process uses many approximations for what's really going on



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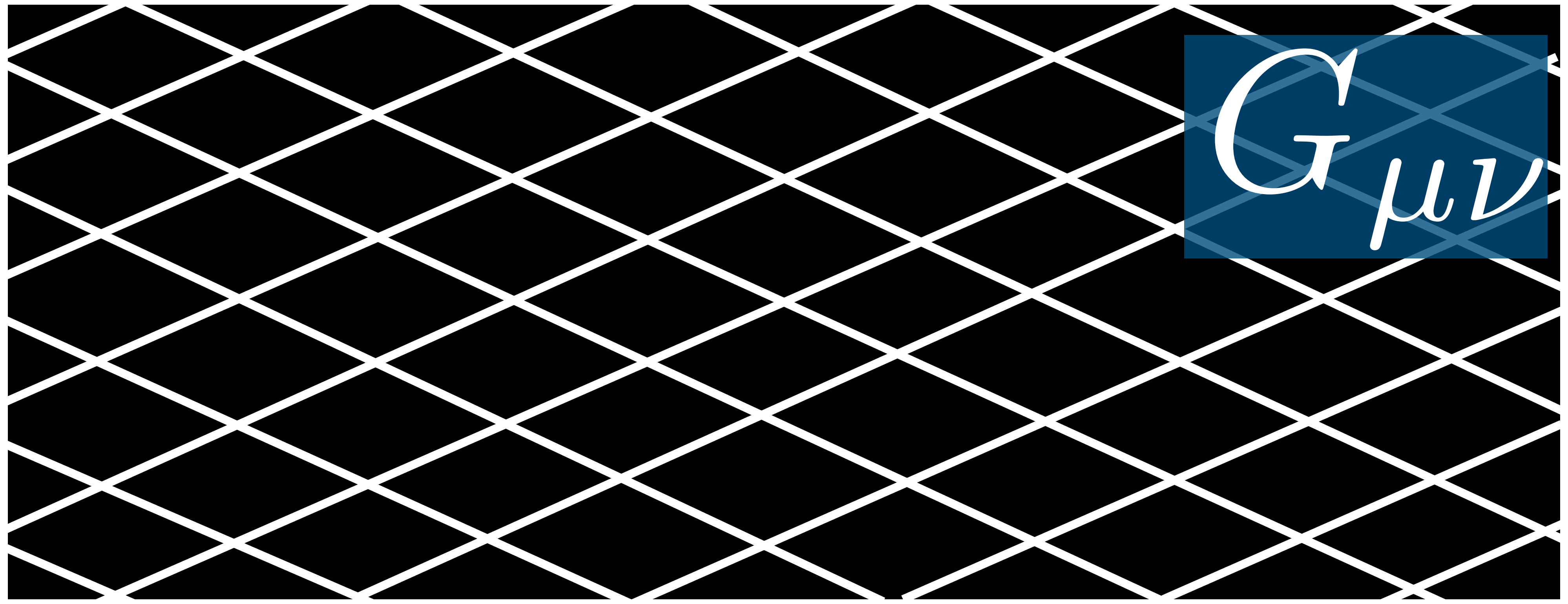

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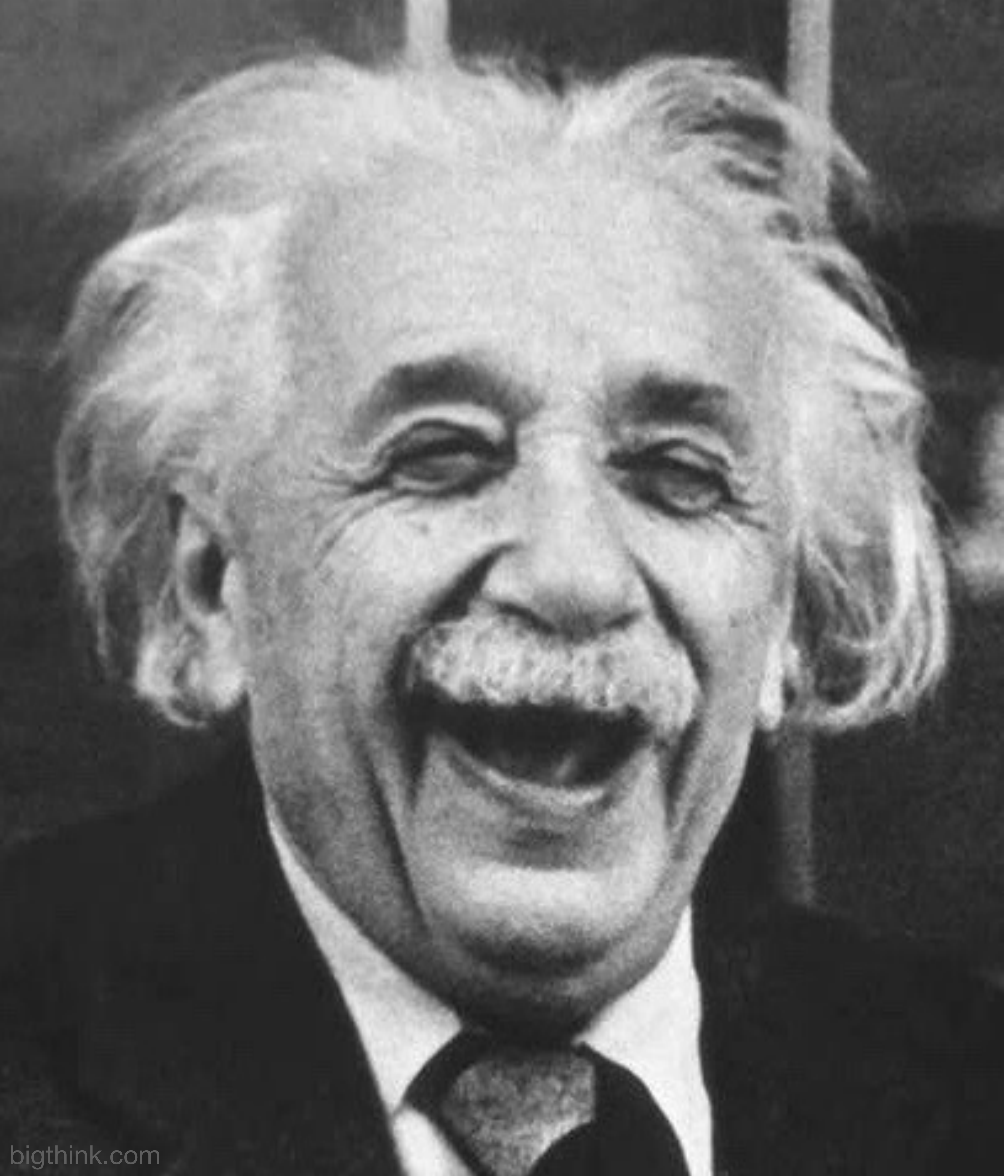
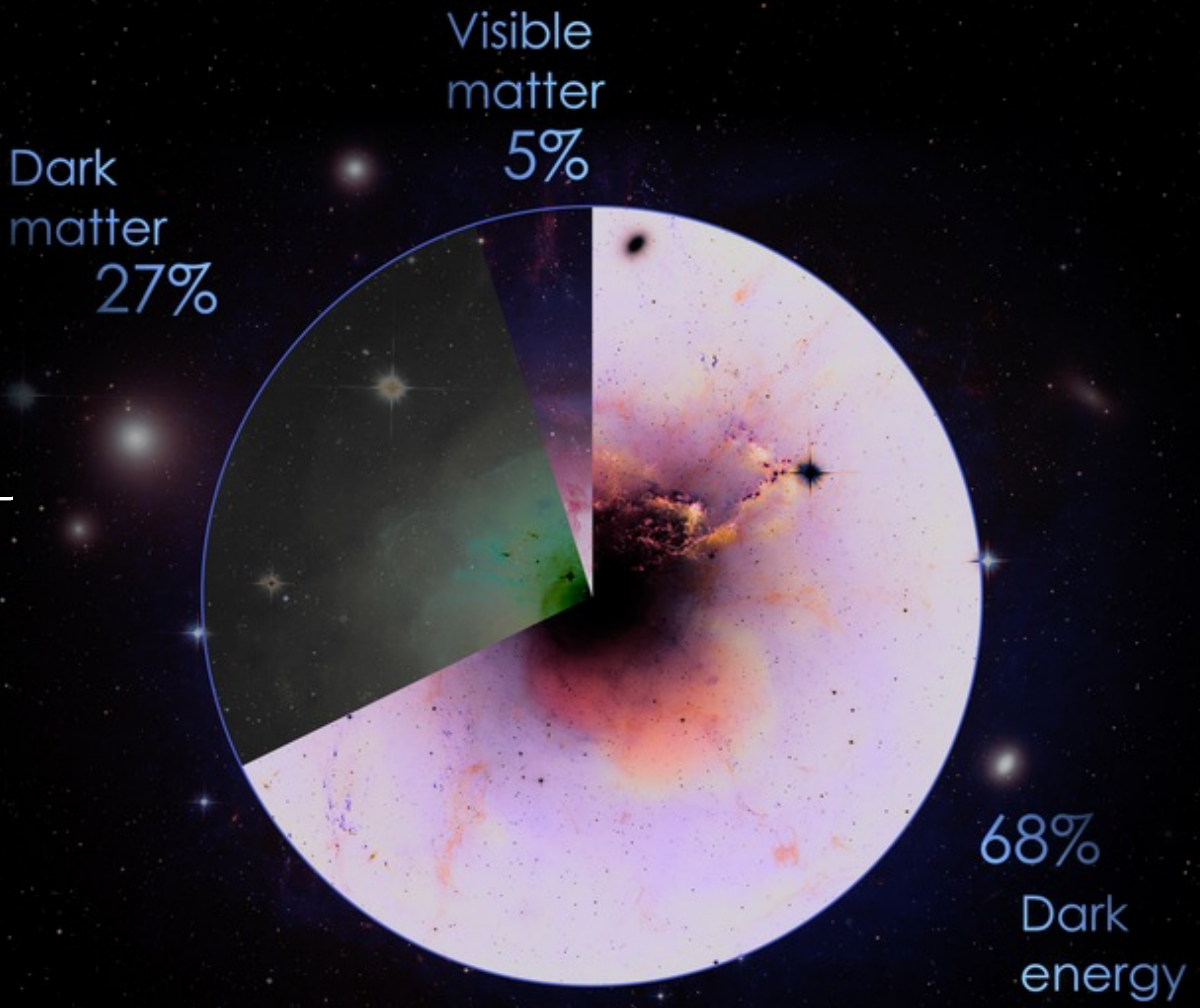


These approximations are justified: Einstein's equations are very difficult to solve

Especially for a highly complex, nonlinear matter distribution

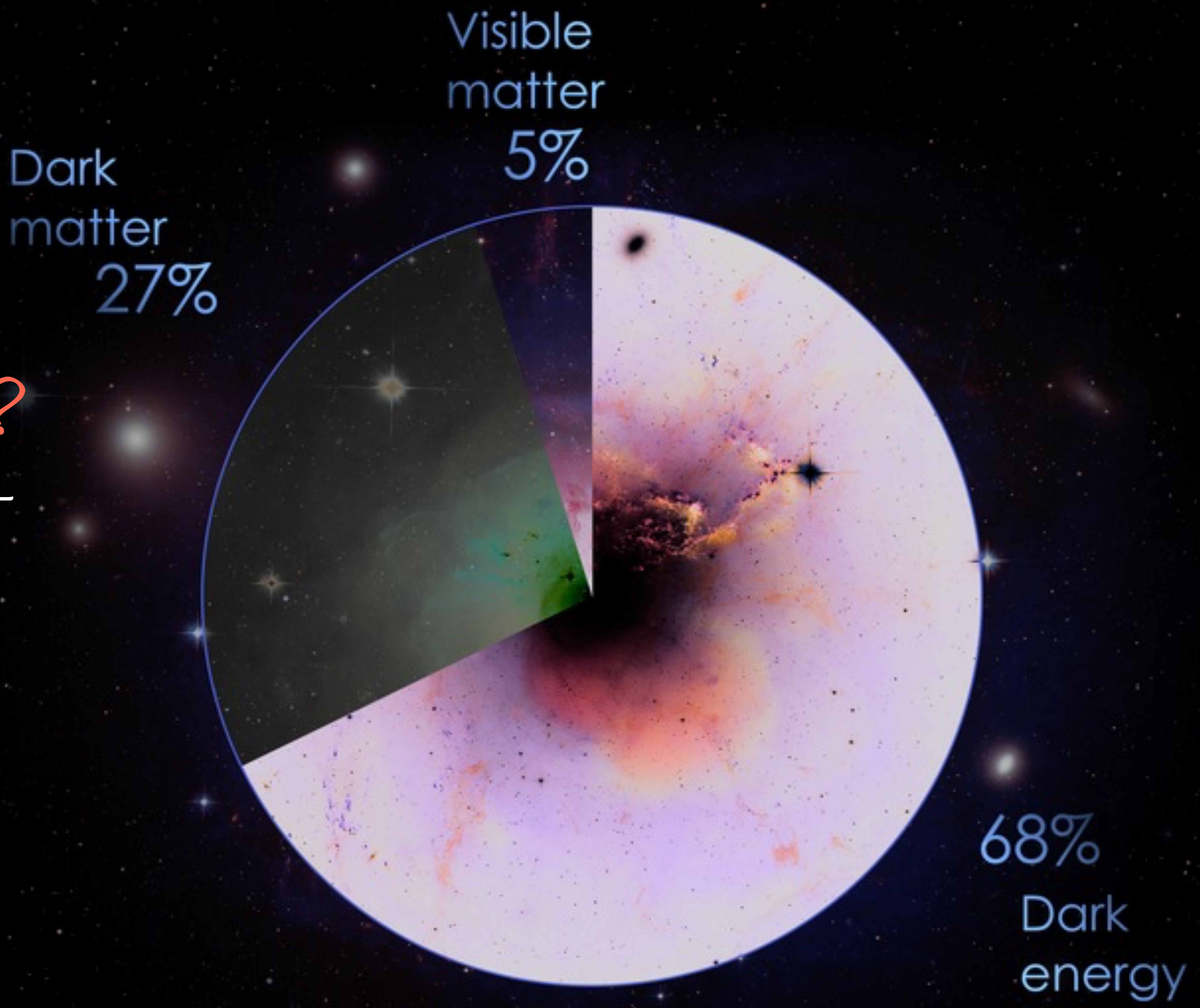
*Our approx. have proven to be very useful, but, our standard model *does* have some issues*

General relativity +
approximations =



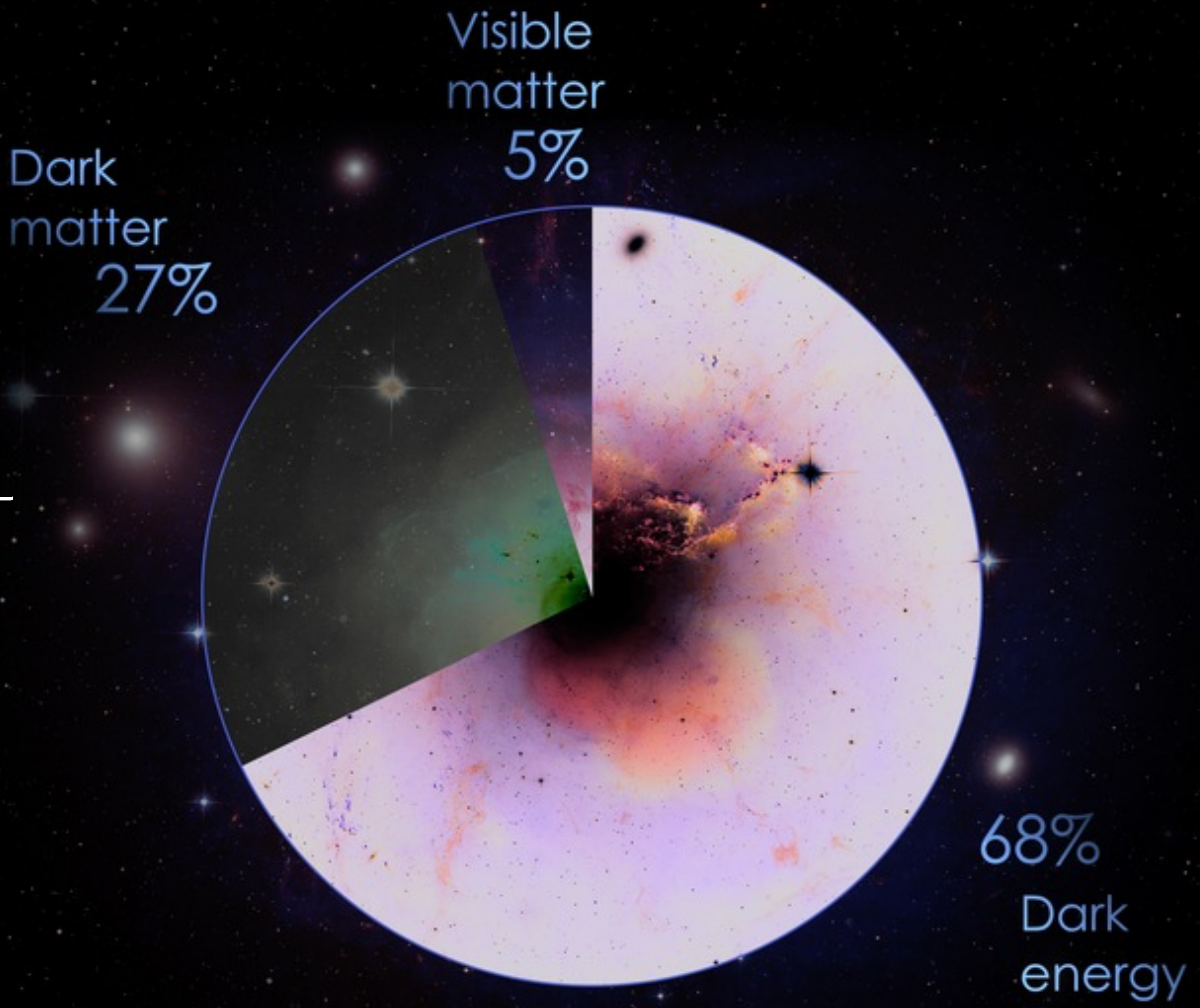
Credit: NASA

General relativity + approximations = ?



Credit: NASA

General relativity + approximations =



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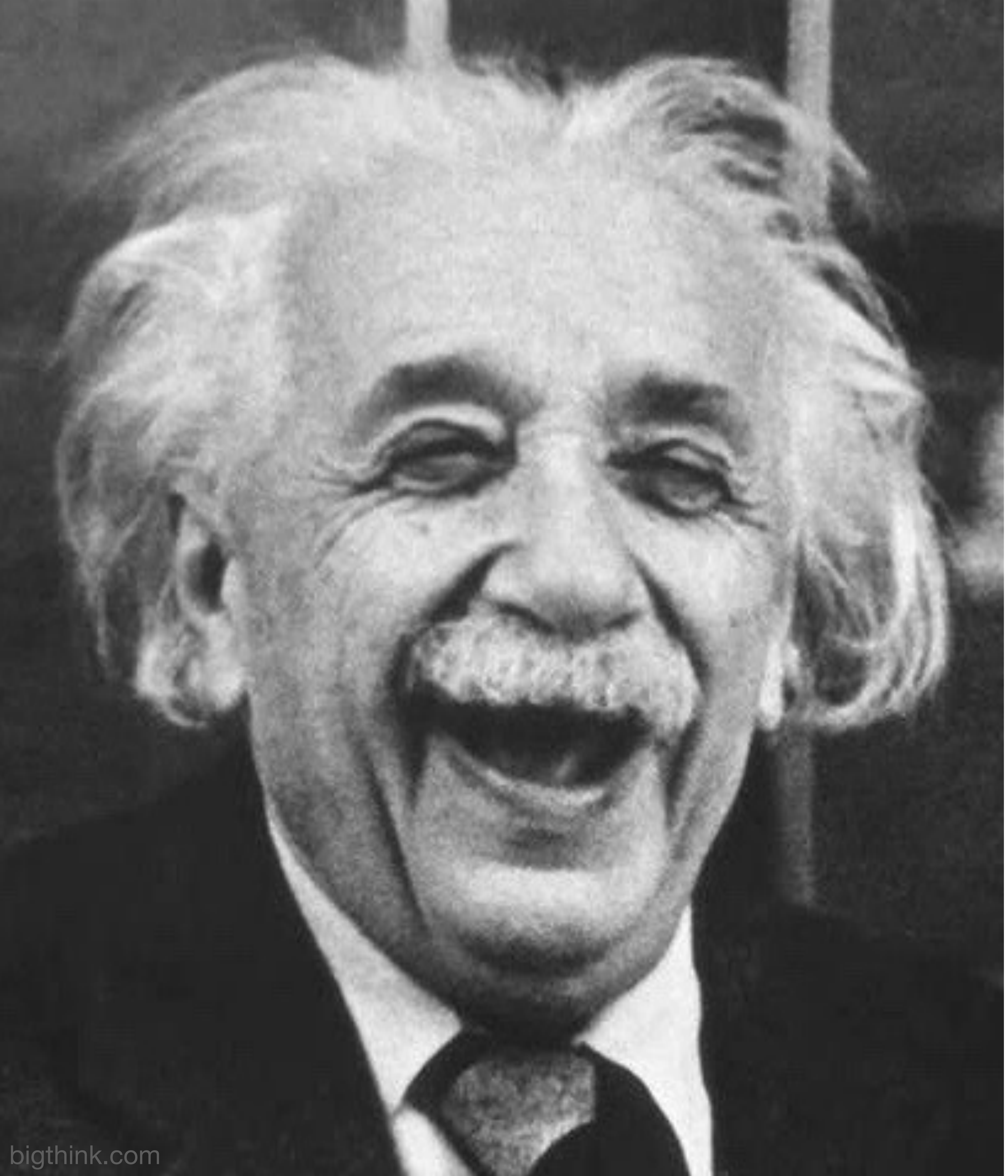

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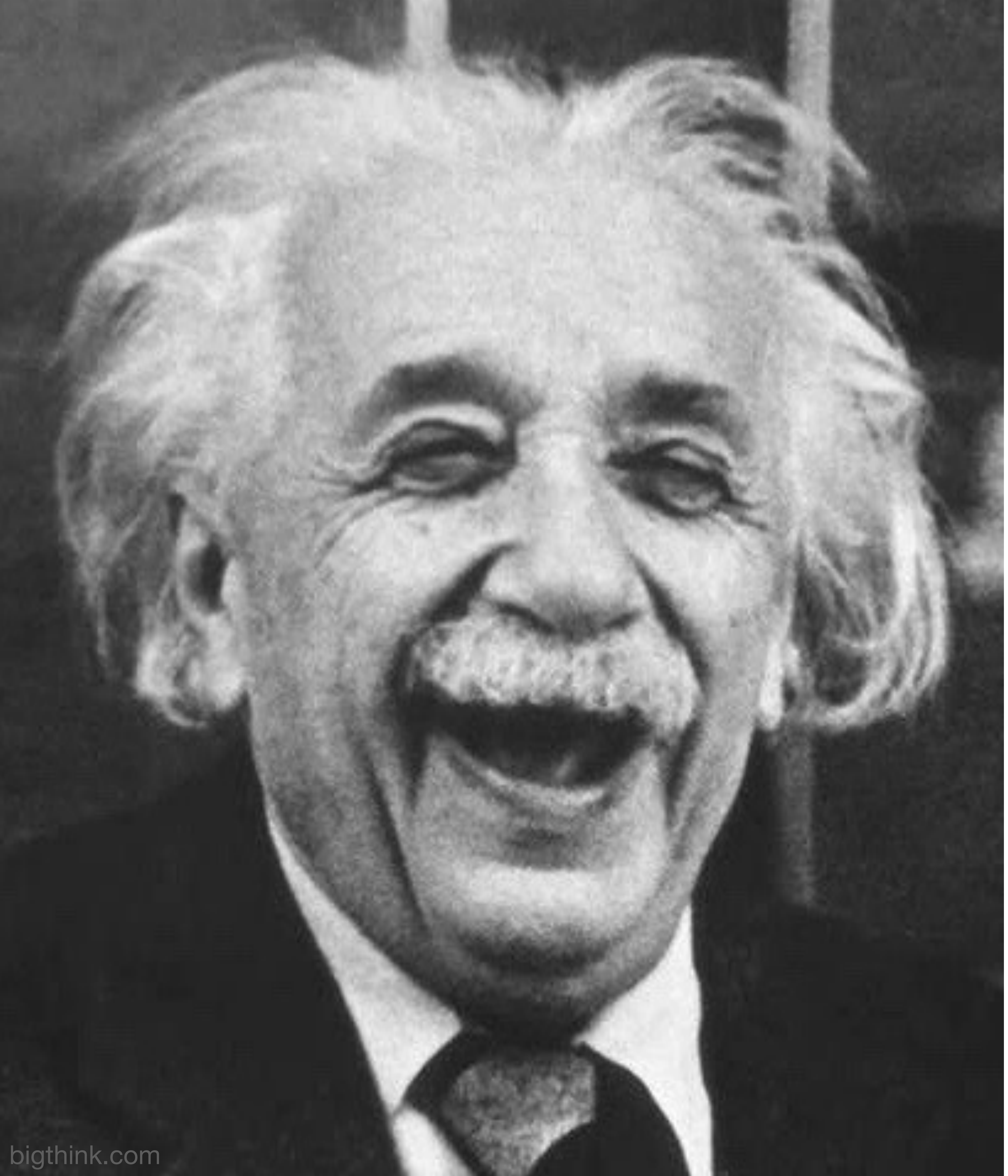

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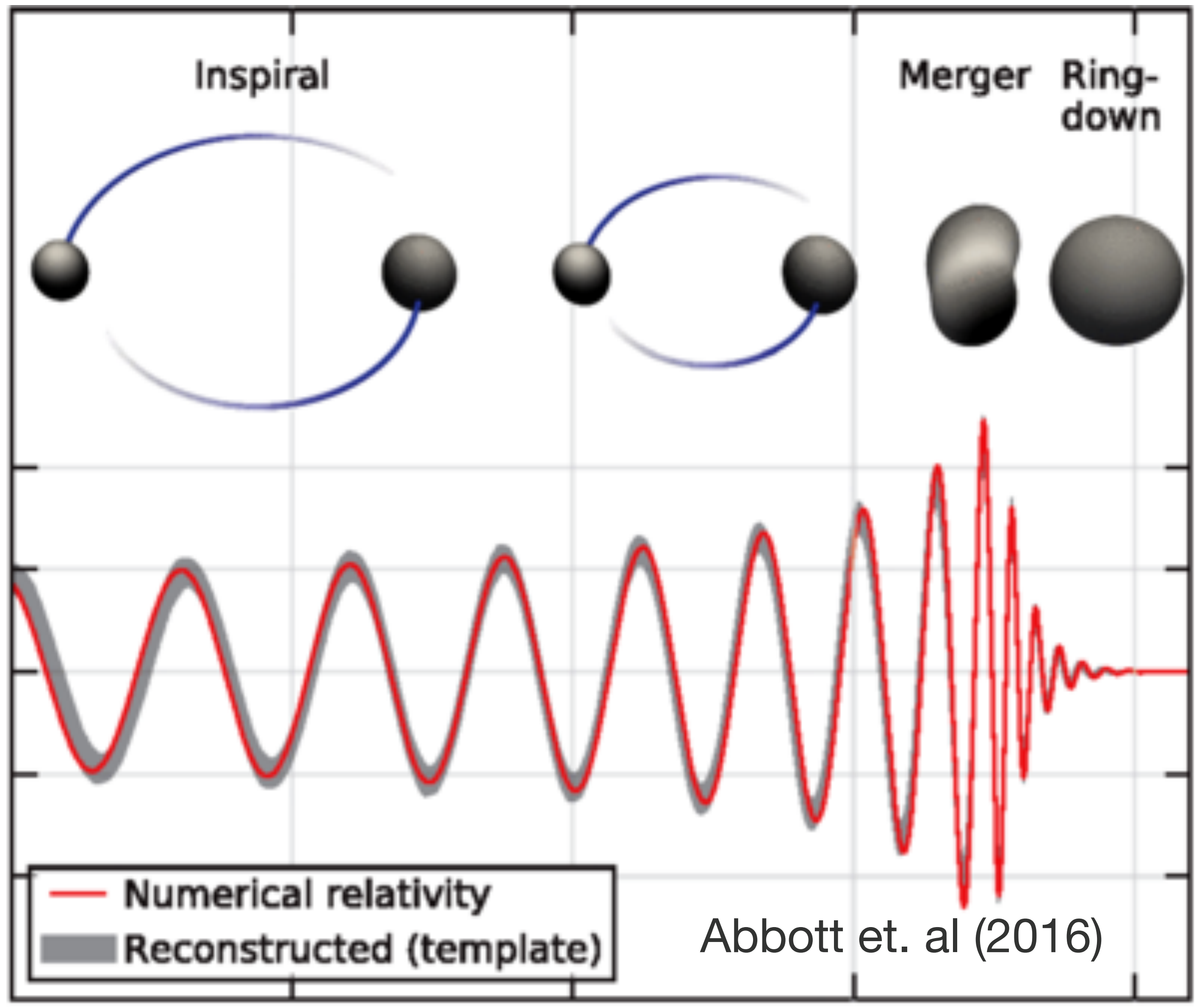
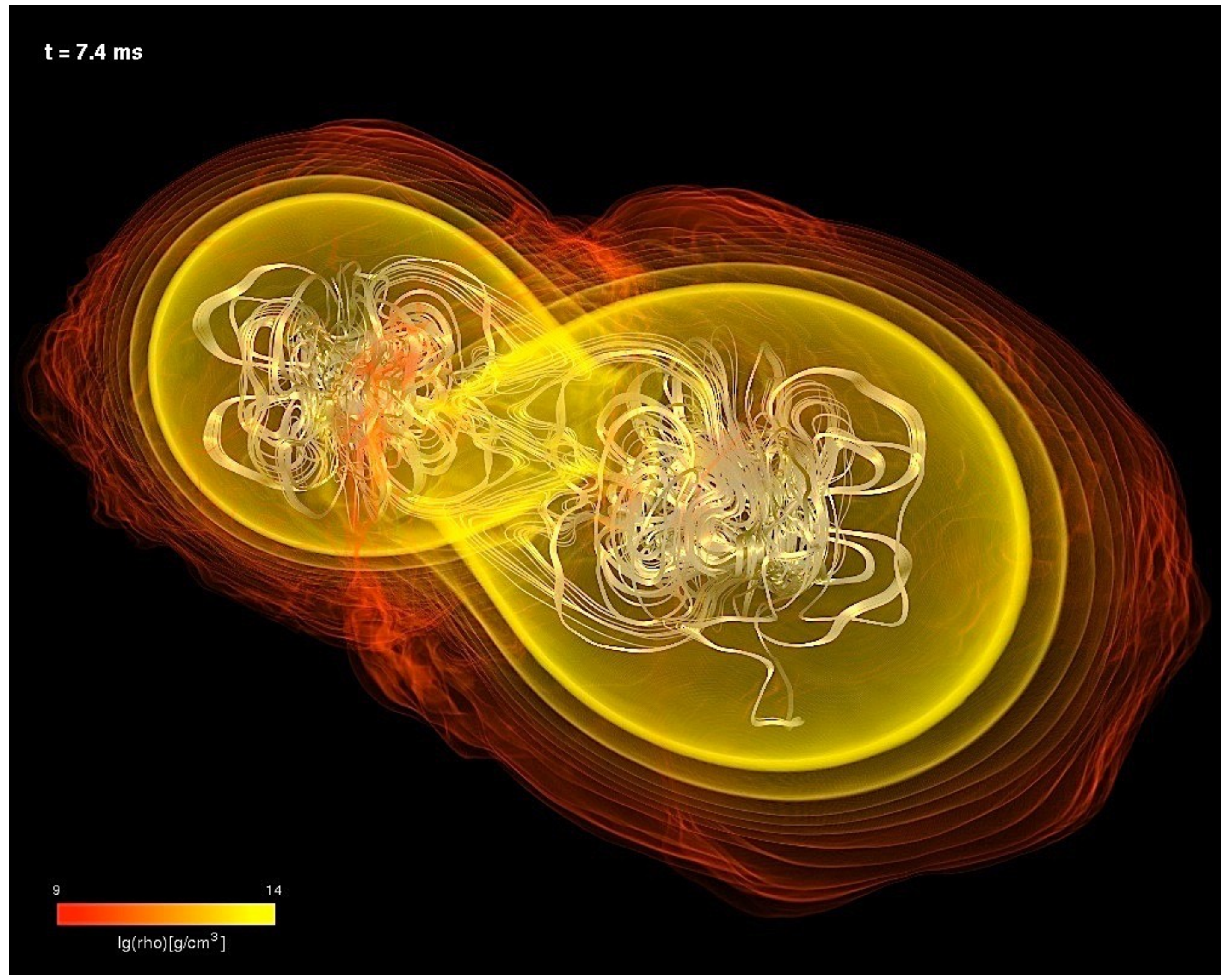
Luckily… we have a way to remove all common physical approximations for GR in cosmology!

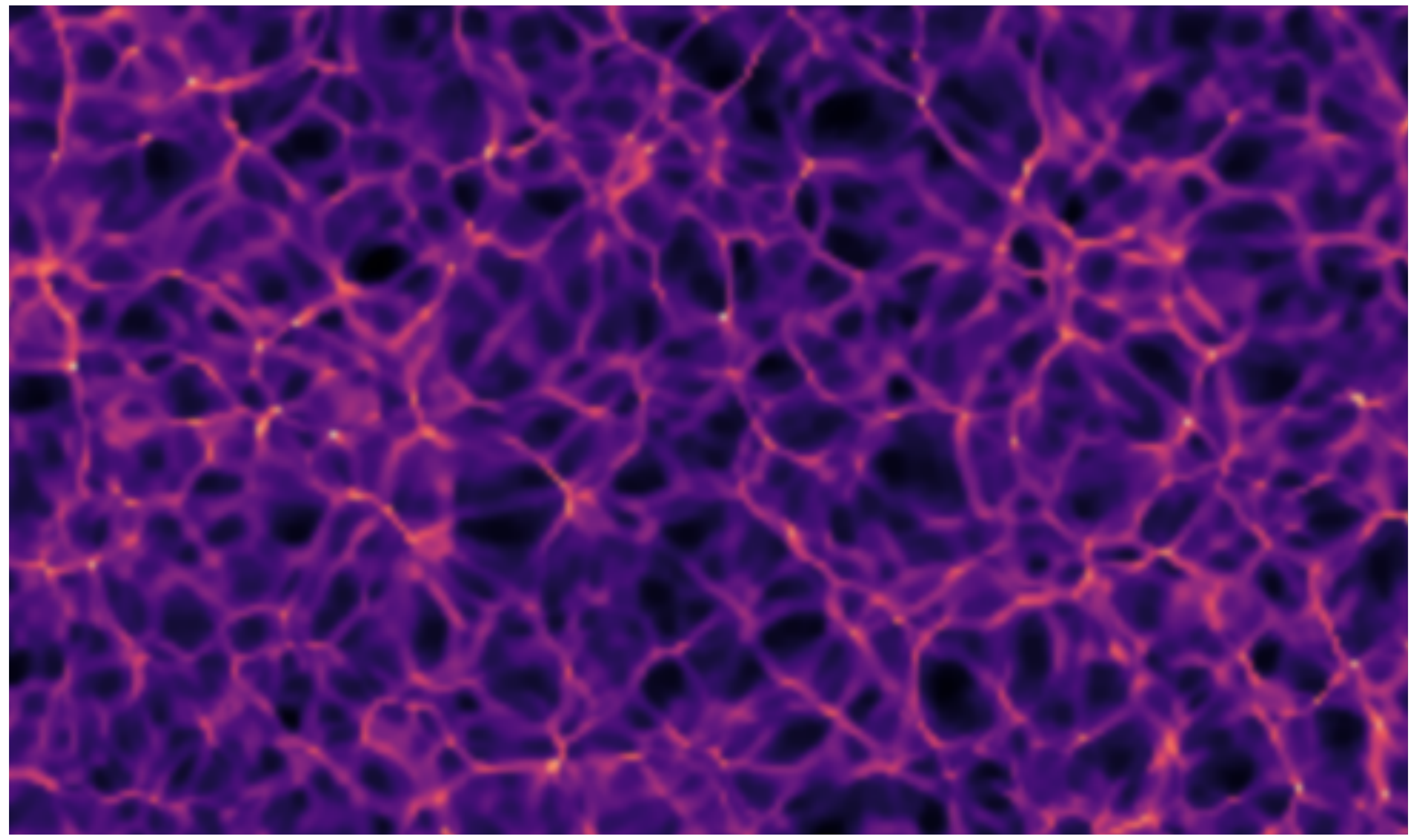
Numerical relativity

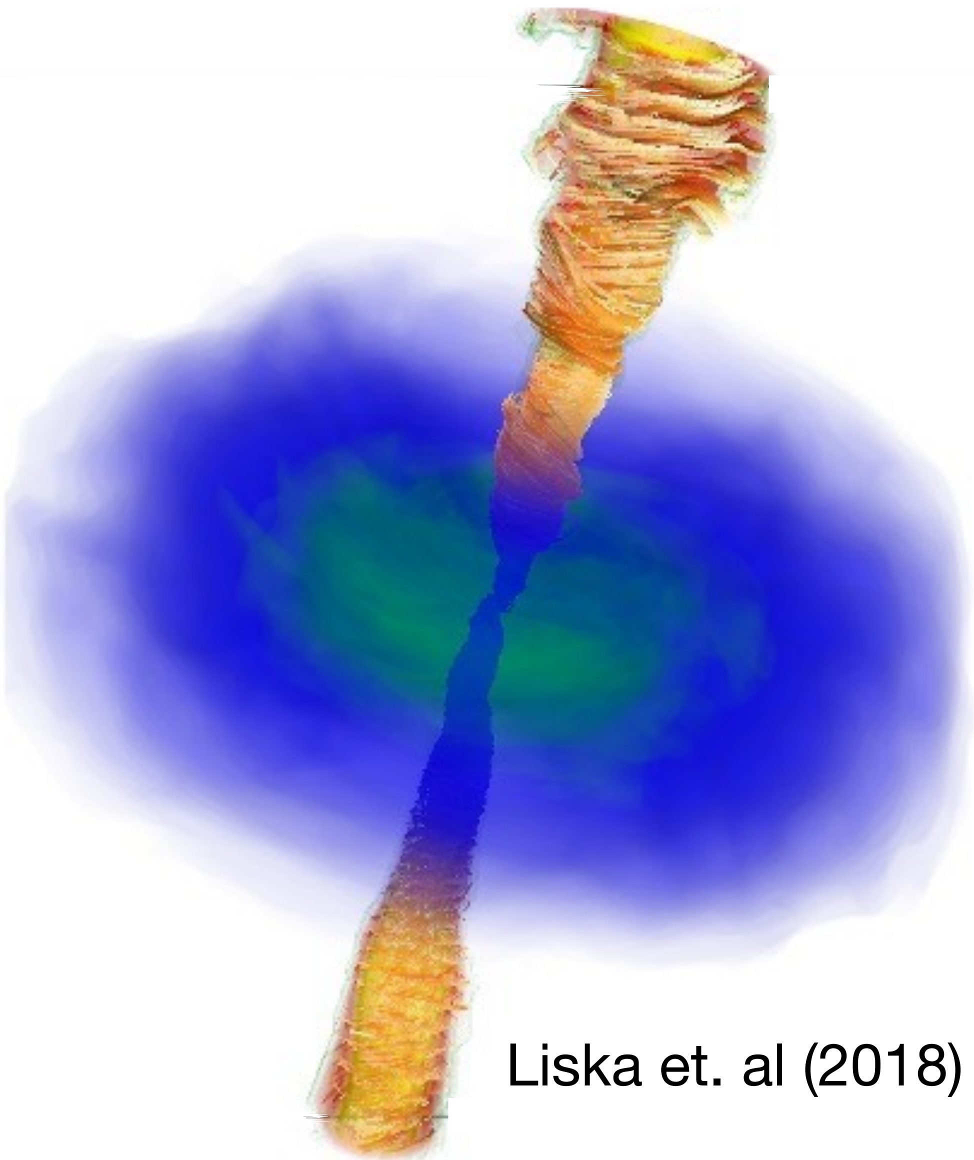


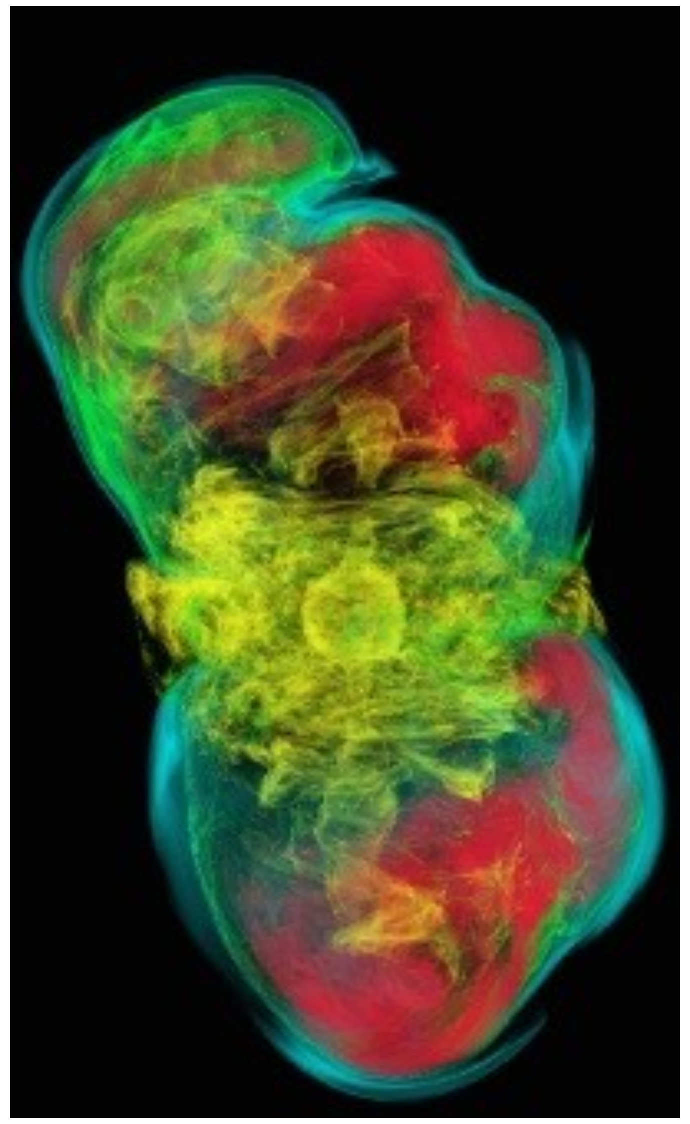
Abbott et. al (2016)



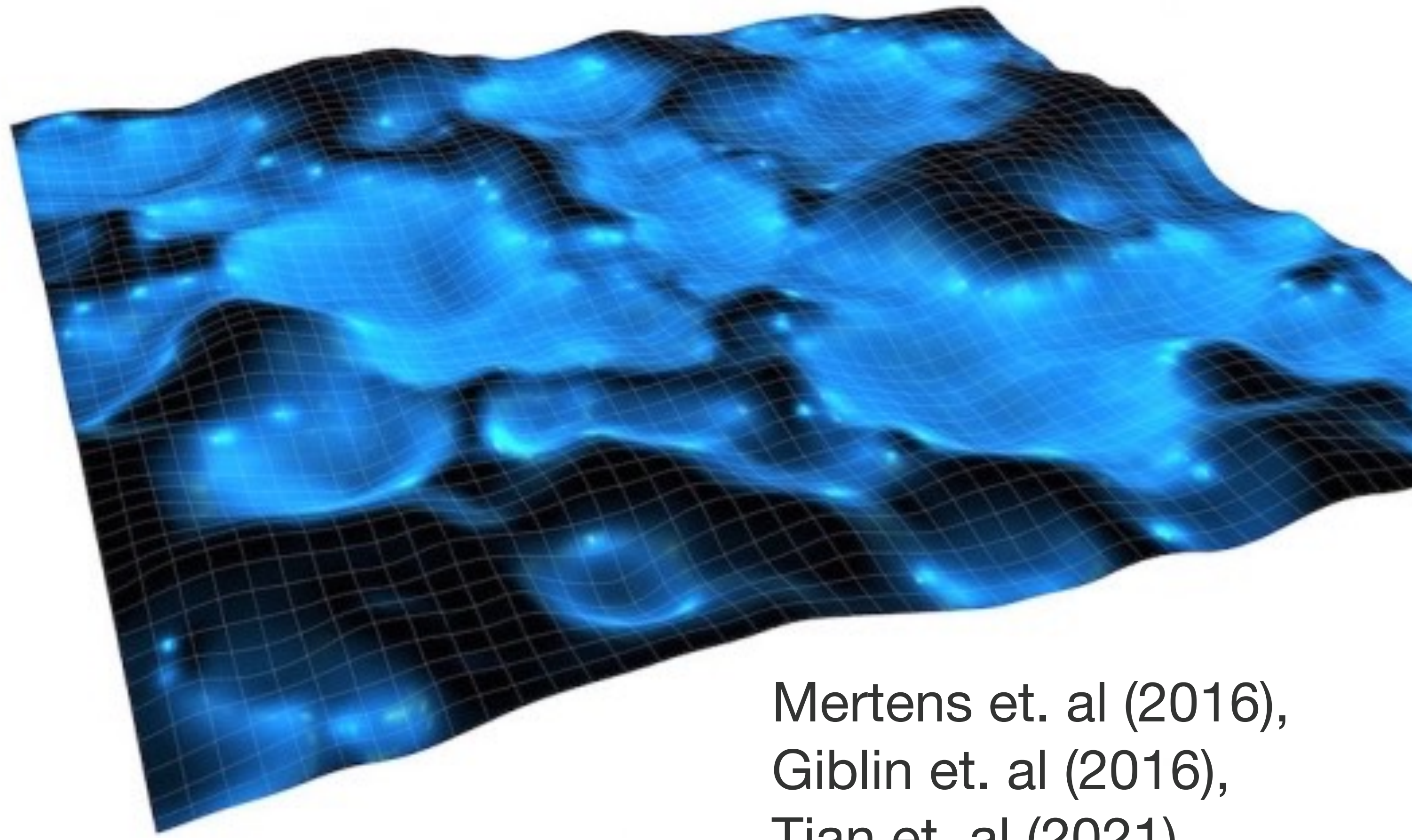
Giacomazzo et. al (2011)


Macpherson et. al (2017-2019)
Macpherson & Heinesen (2021)
Heinesen & Macpherson (2022)


Liska et. al (2018)


Moesta et. al (2014)

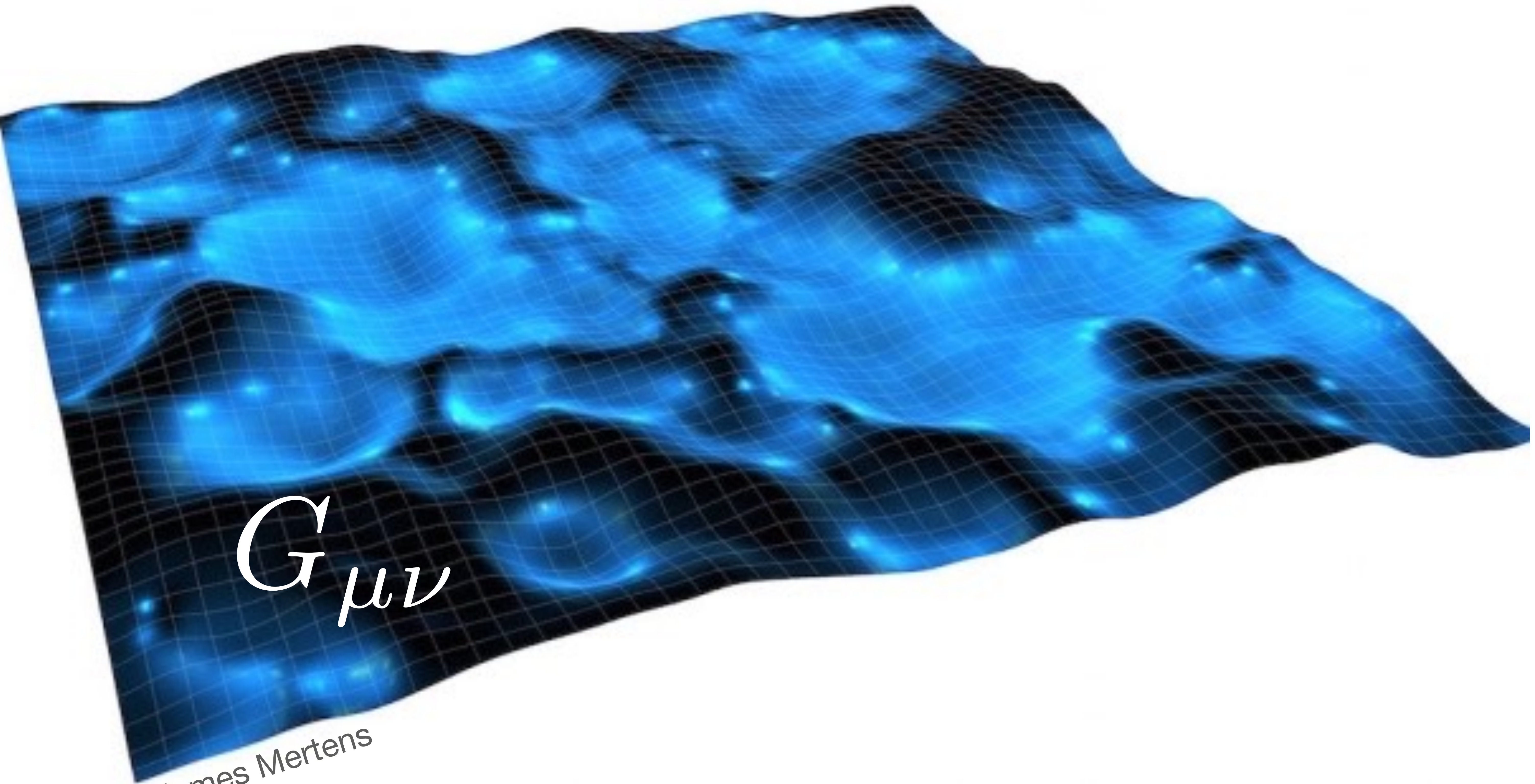
Allows us to remove common simplifying assumptions about gravity and geometry by solving Einstein's equations *directly*


Mertens et. al (2016),
Giblin et. al (2016),
Tian et. al (2021)

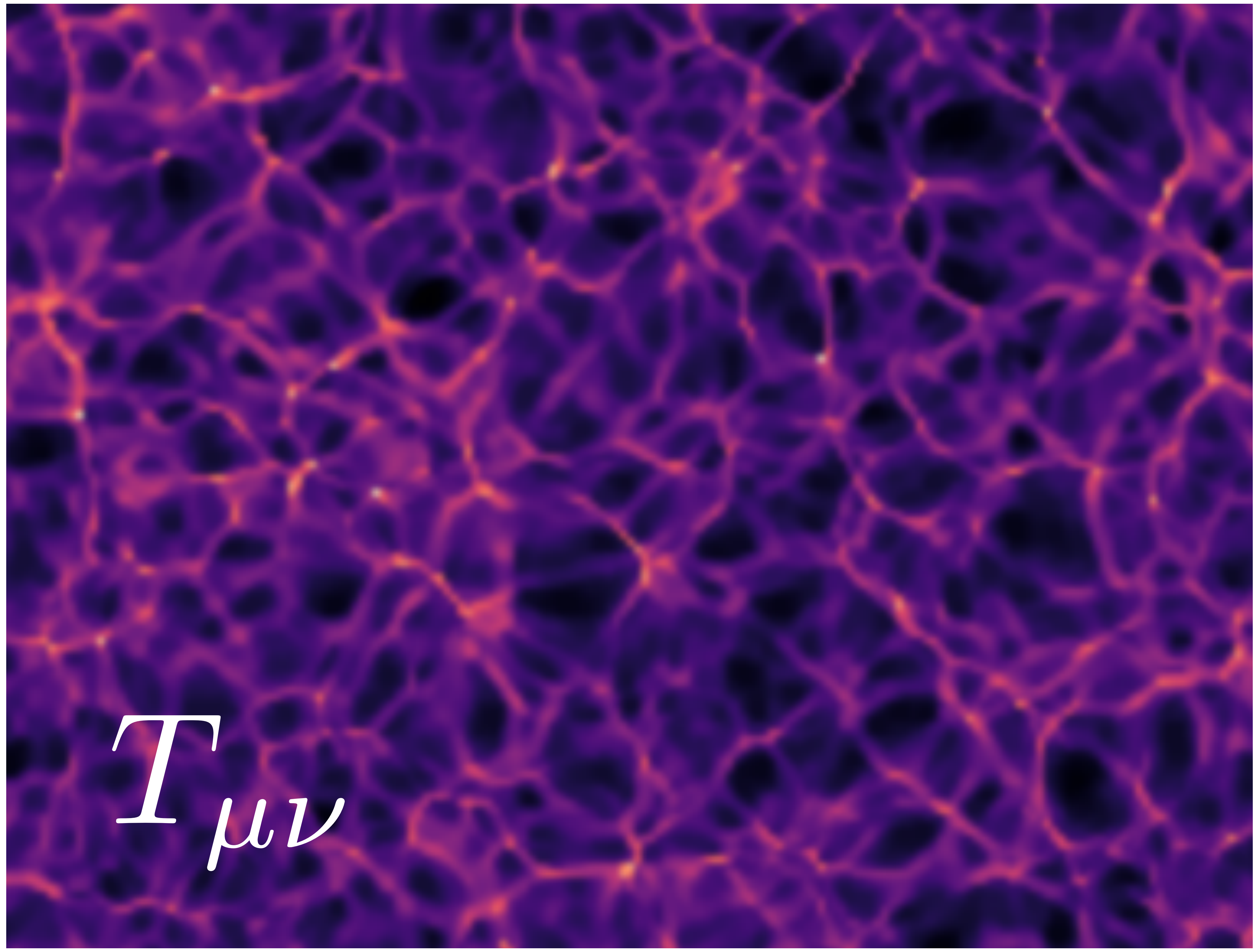
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In N-body simulations, the left hand side is separated from the right hand side

Numerical relativity allows us to maintain communication between the left and right hand sides, thus achieve nonlinearity of *both matter and space-time*

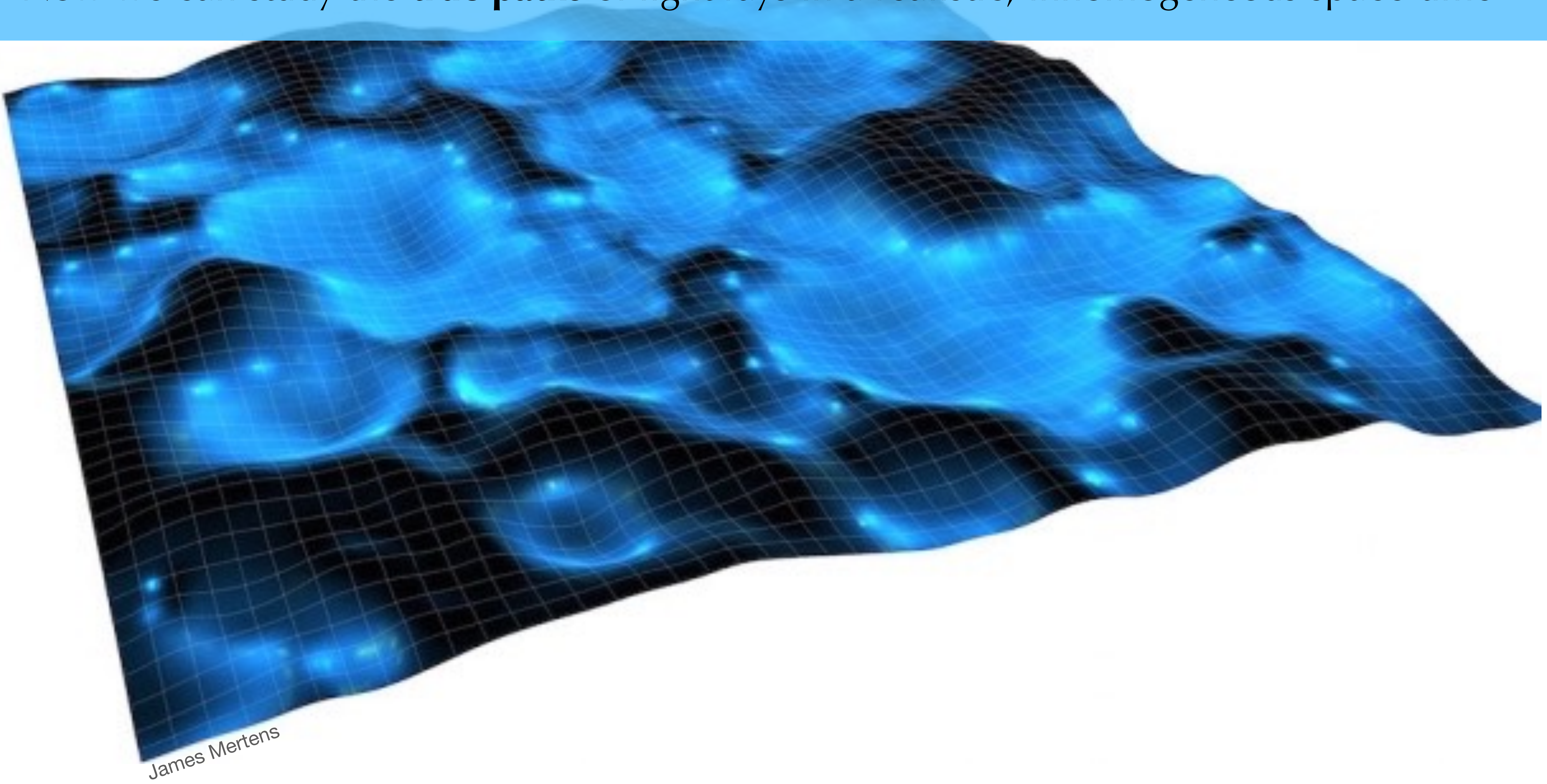


James Mertens



Macpherson et. al (2019)

Now we can study the **true paths** of light rays in a realistic, inhomogeneous space-time



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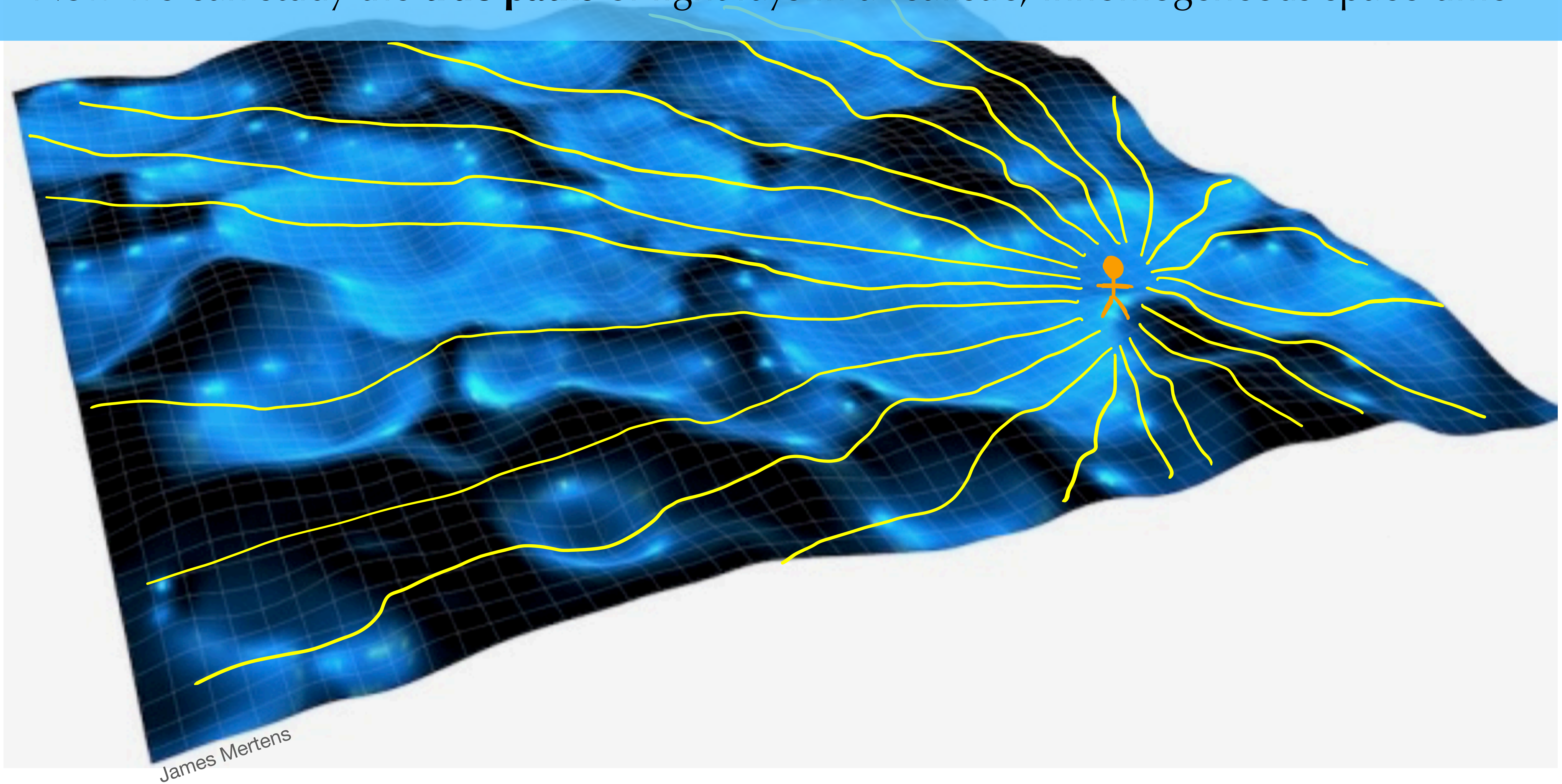

See Macpherson (2023) for details

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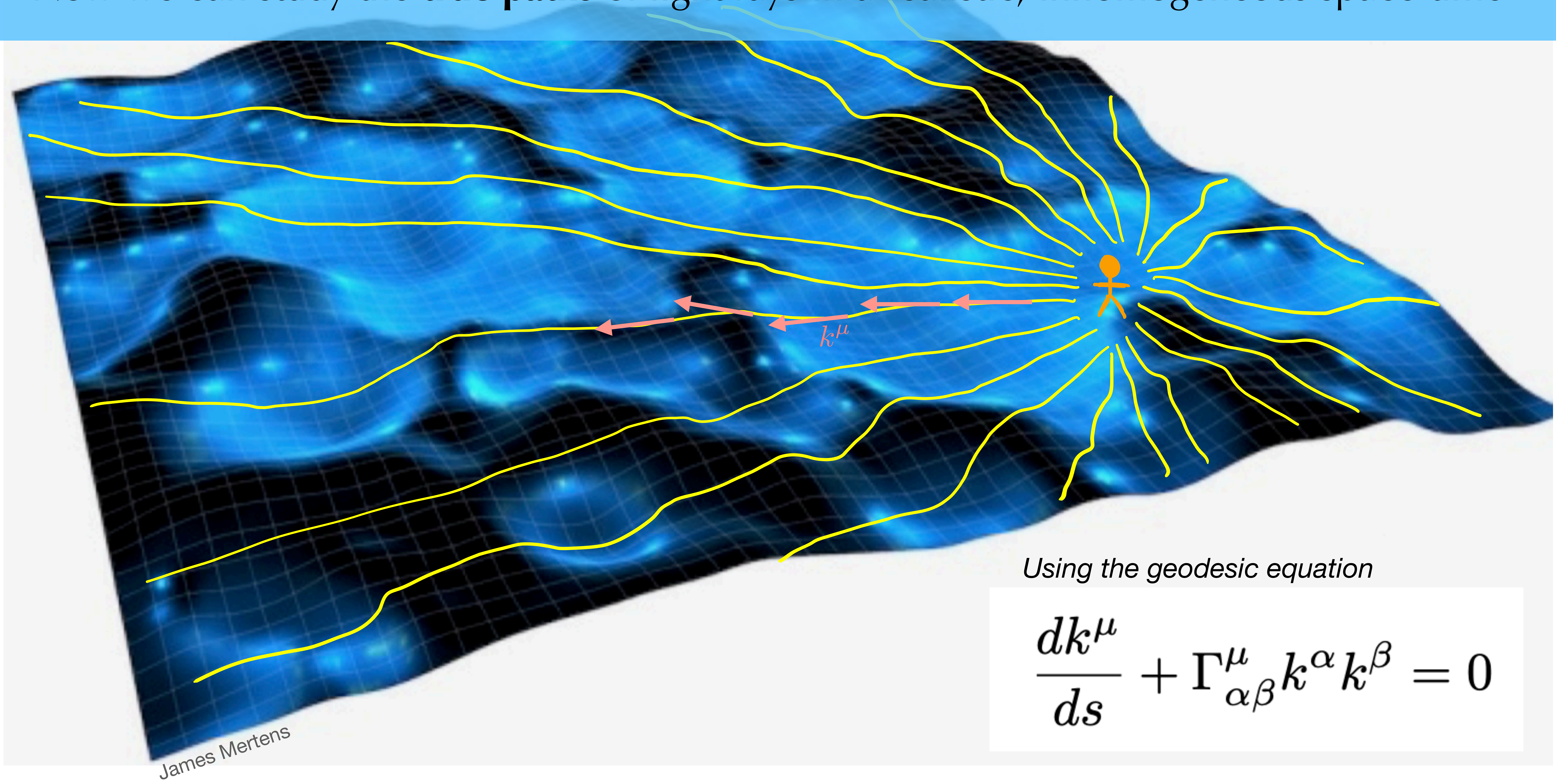


Using the geodesic equation

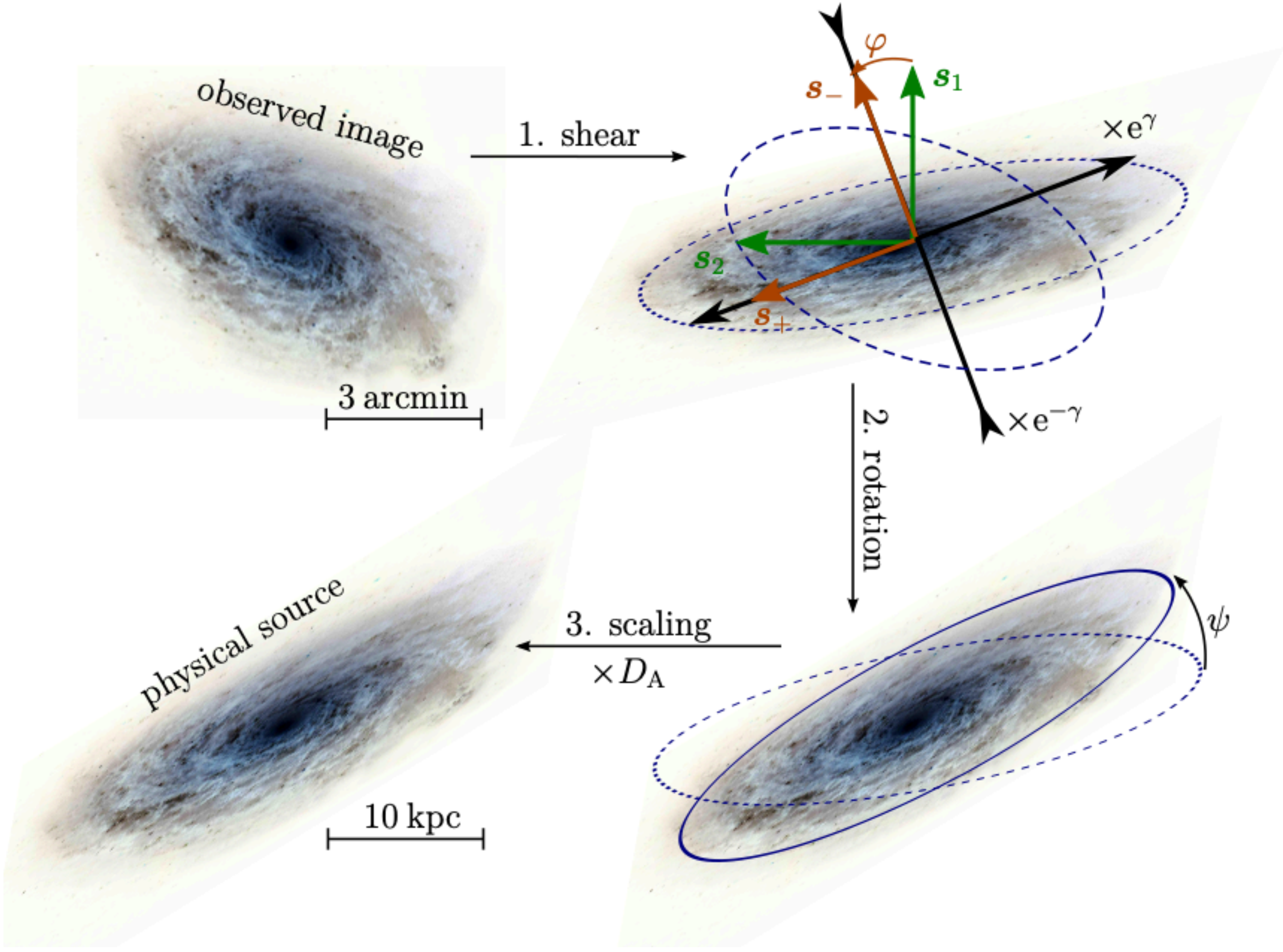
$$\frac{dk^\mu}{ds} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0$$

See Macpherson (2023) for details

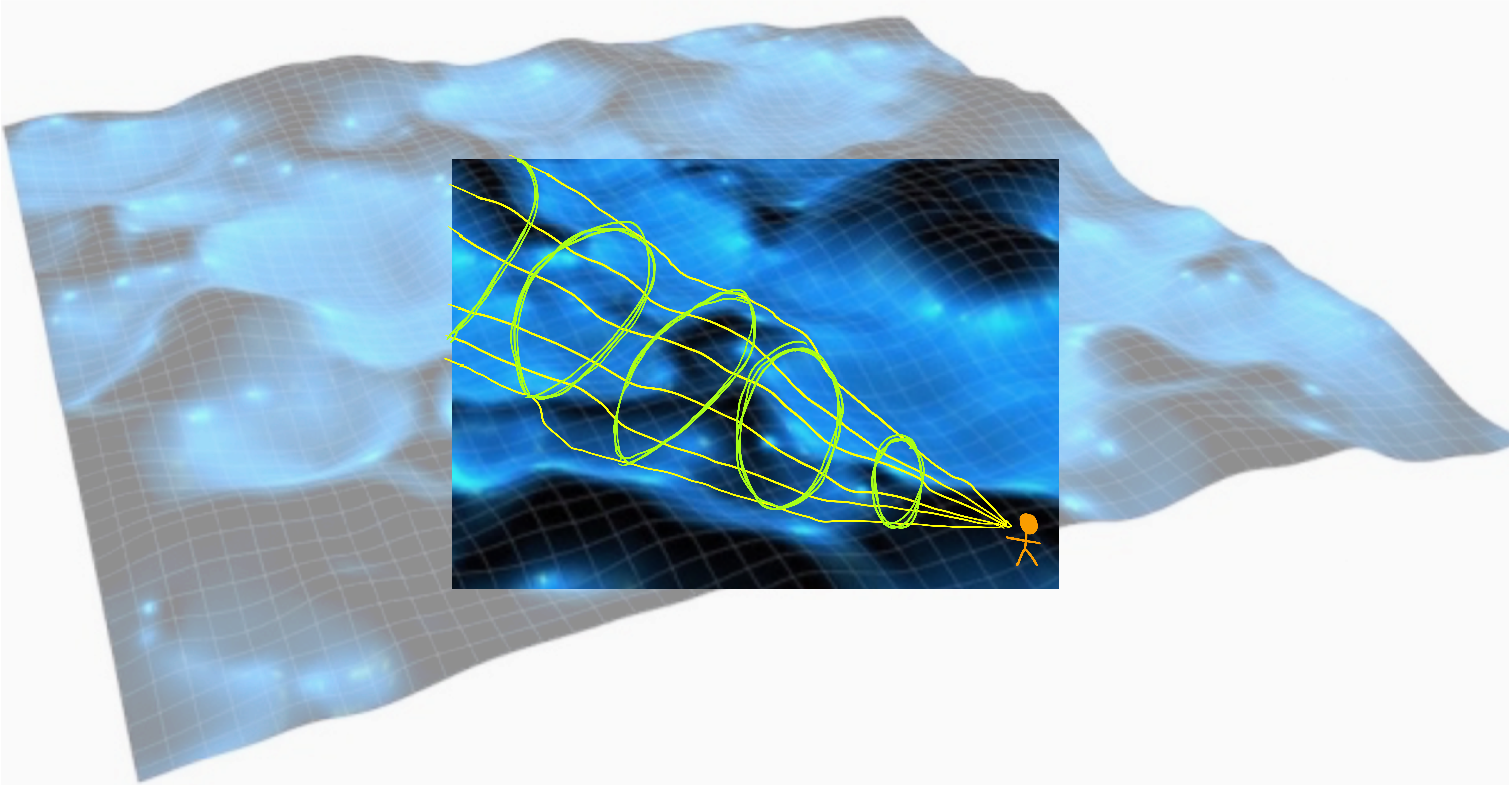
Gravitational lensing

There are several impacts of weak lensing on the images we observe

*One of them is the **convergence**, which is essentially a magnification of sources w.r.t their true distance*

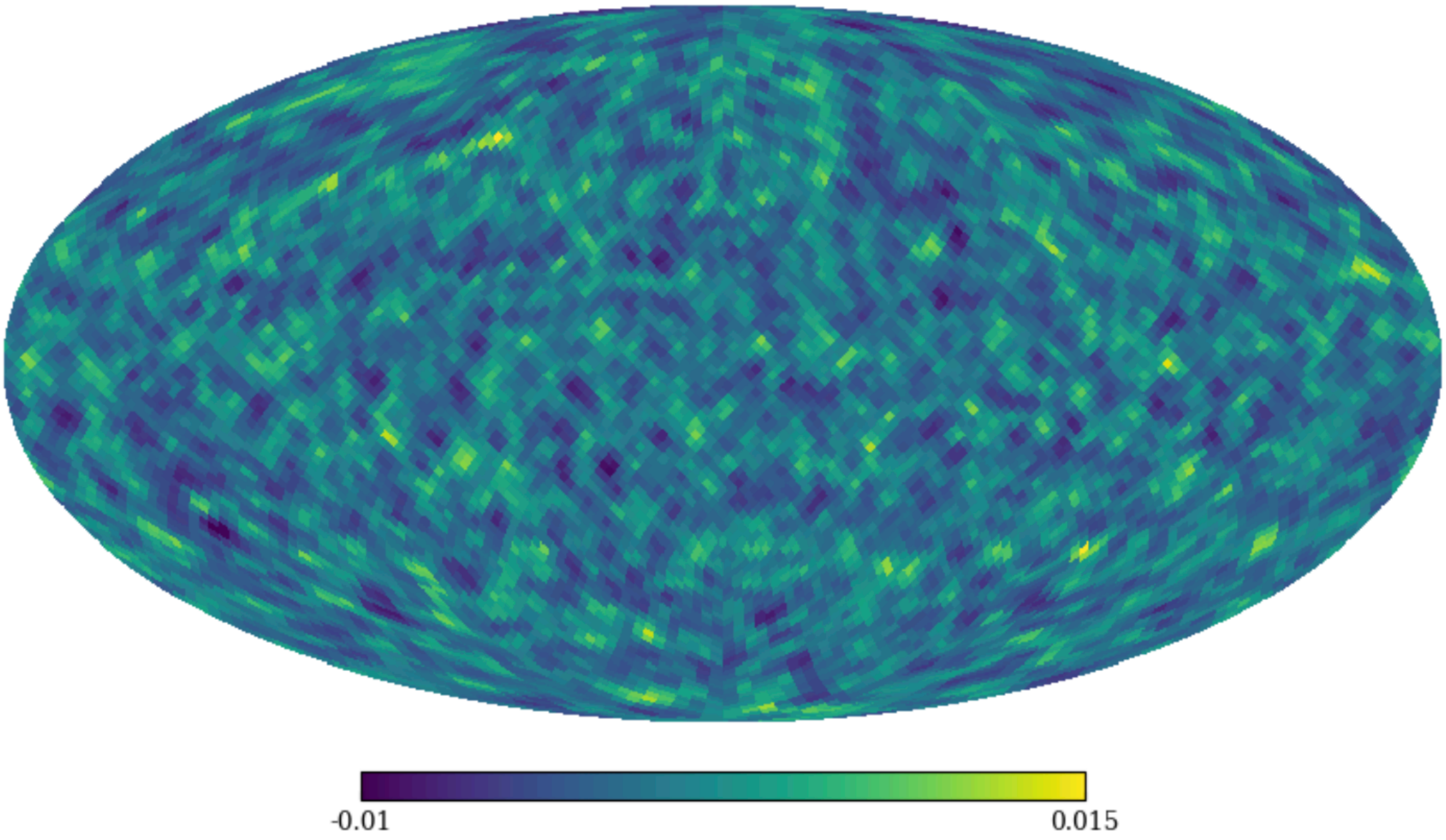


Fleury 2015 (PhD thesis)

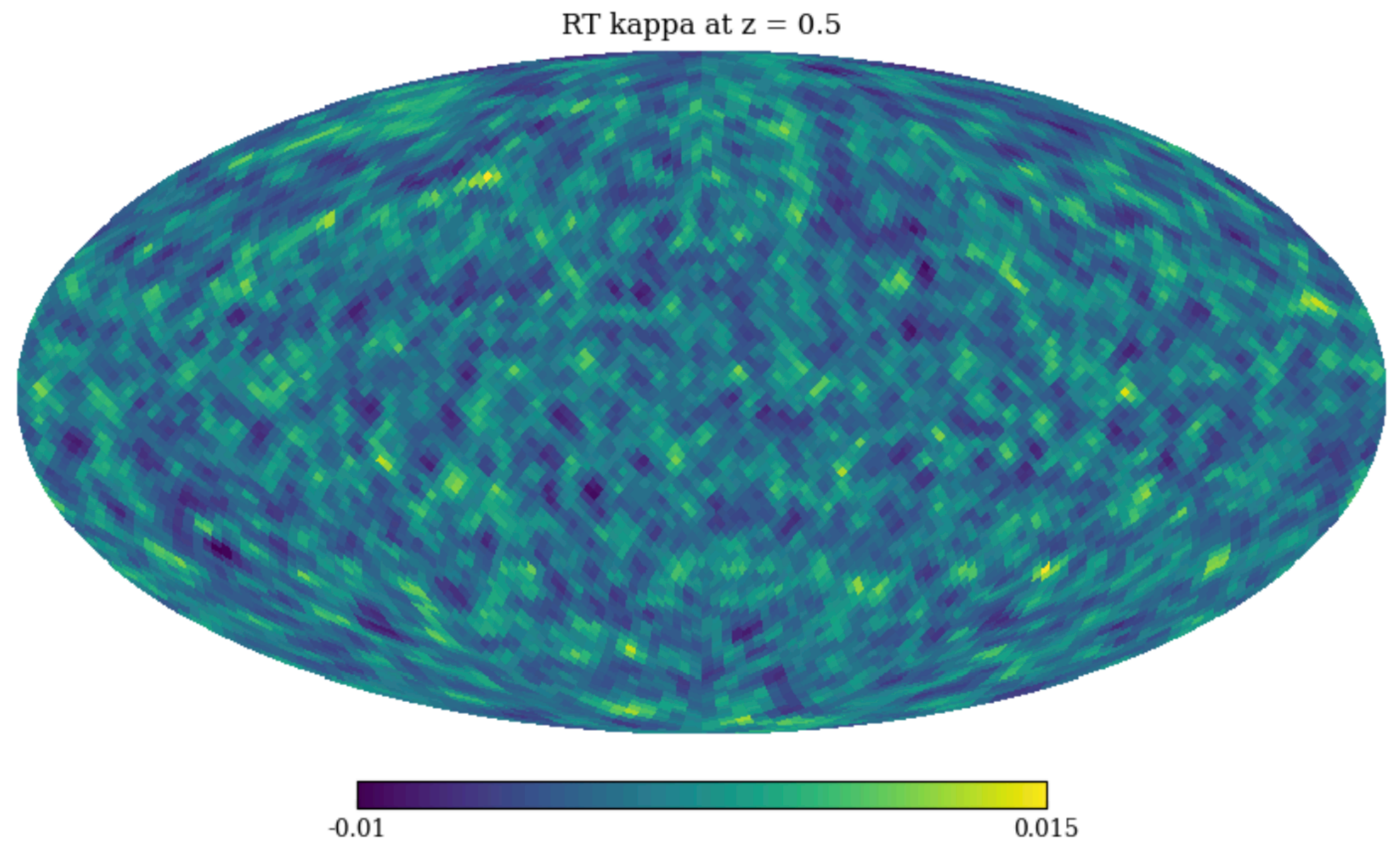


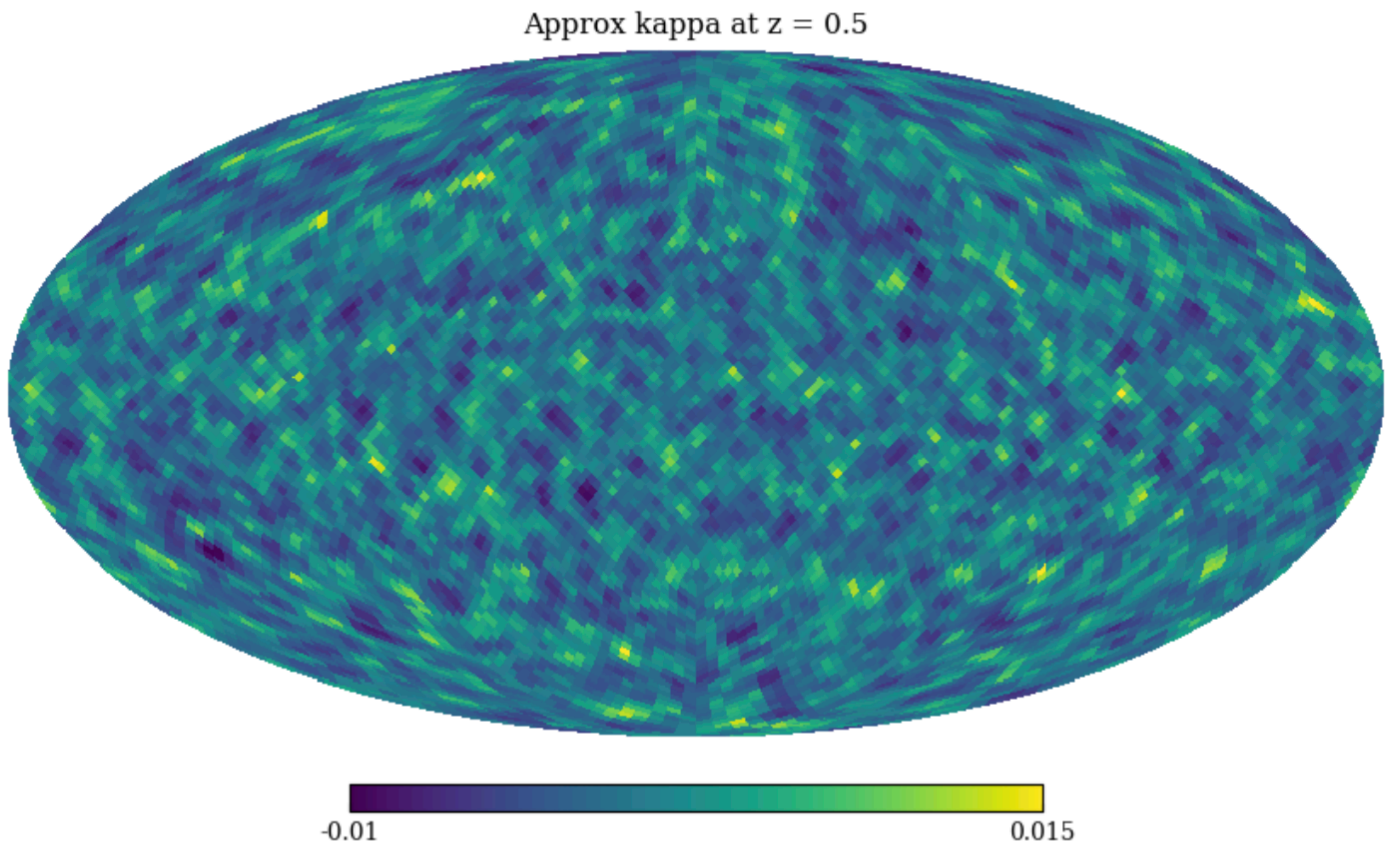
See Macpherson (2023) for details

Lensing convergence at redshift $z = 0.5$ in a numerical relativity simulation



Macpherson (2024; in prep)





We want to compare this lensing map in full GR (top) to some kind of commonly-used approximation

It is very common to use an integral of the density field in the simulation along the line of sight. This gives the lower map.

*They look very similar by eye, but there is a *small* difference in the power spectra*

Macpherson (2024; in prep)

Numerical relativity simulations of large-scale structure formation give us a cosmic web *as well as* all of the information about the underlying geometry

We can use these simulations to stress test the founding assumptions of standard cosmology

We can also use them to study general relativistic observables in an assumption-free framework

We are working on comparing the weak lensing convergence signal in an NR simulation to a widely-used approximation

While there is a small difference - I am currently doing thorough tests to see if this result is robust

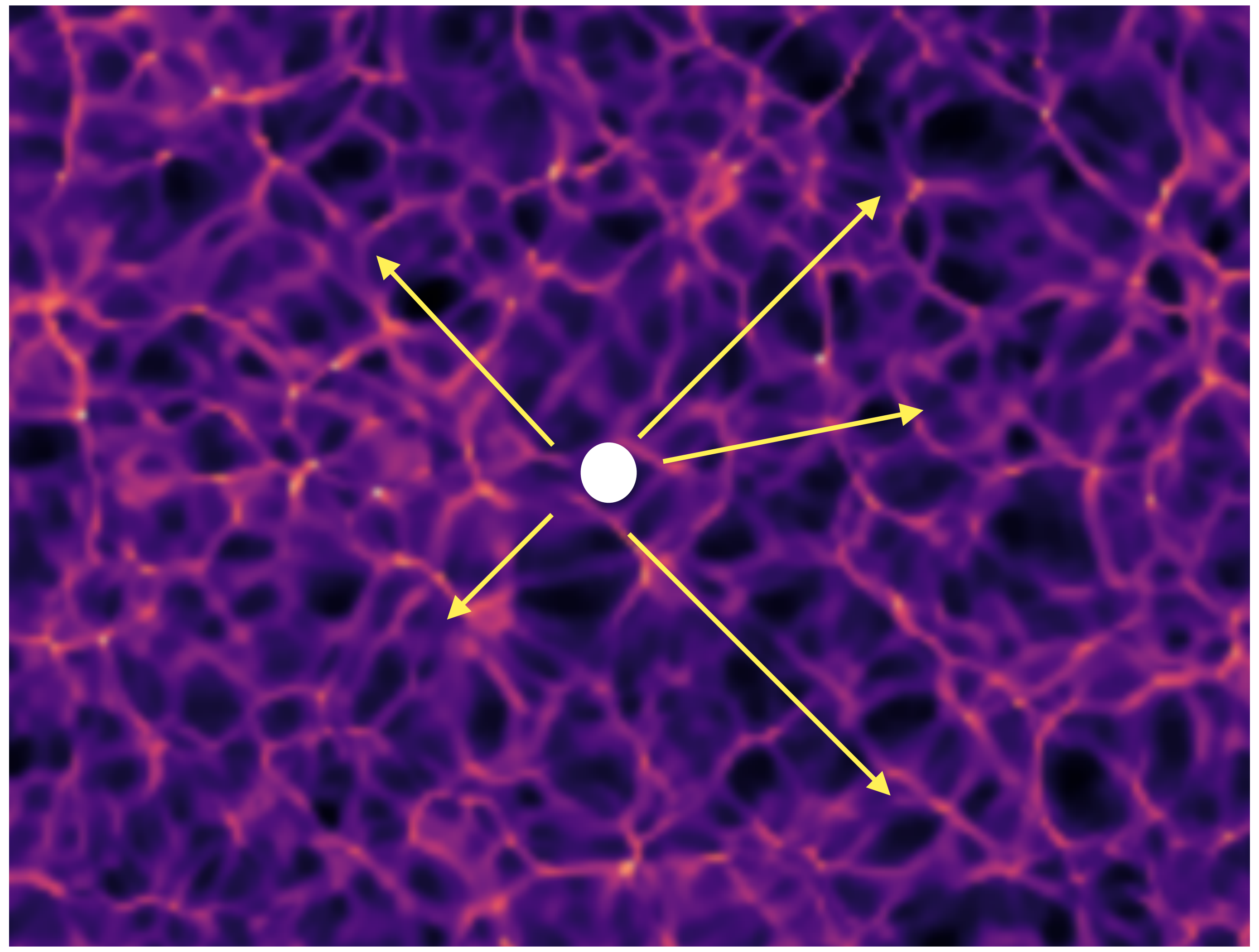
Stay tuned!

Extra stuff

Raytracing in full GR

$$\frac{dk^{\mu}}{ds} + \Gamma^{\mu}_{\alpha\beta} k^{\alpha} k^{\beta} = 0$$

✓ Trace the path of geodesics through the simulation

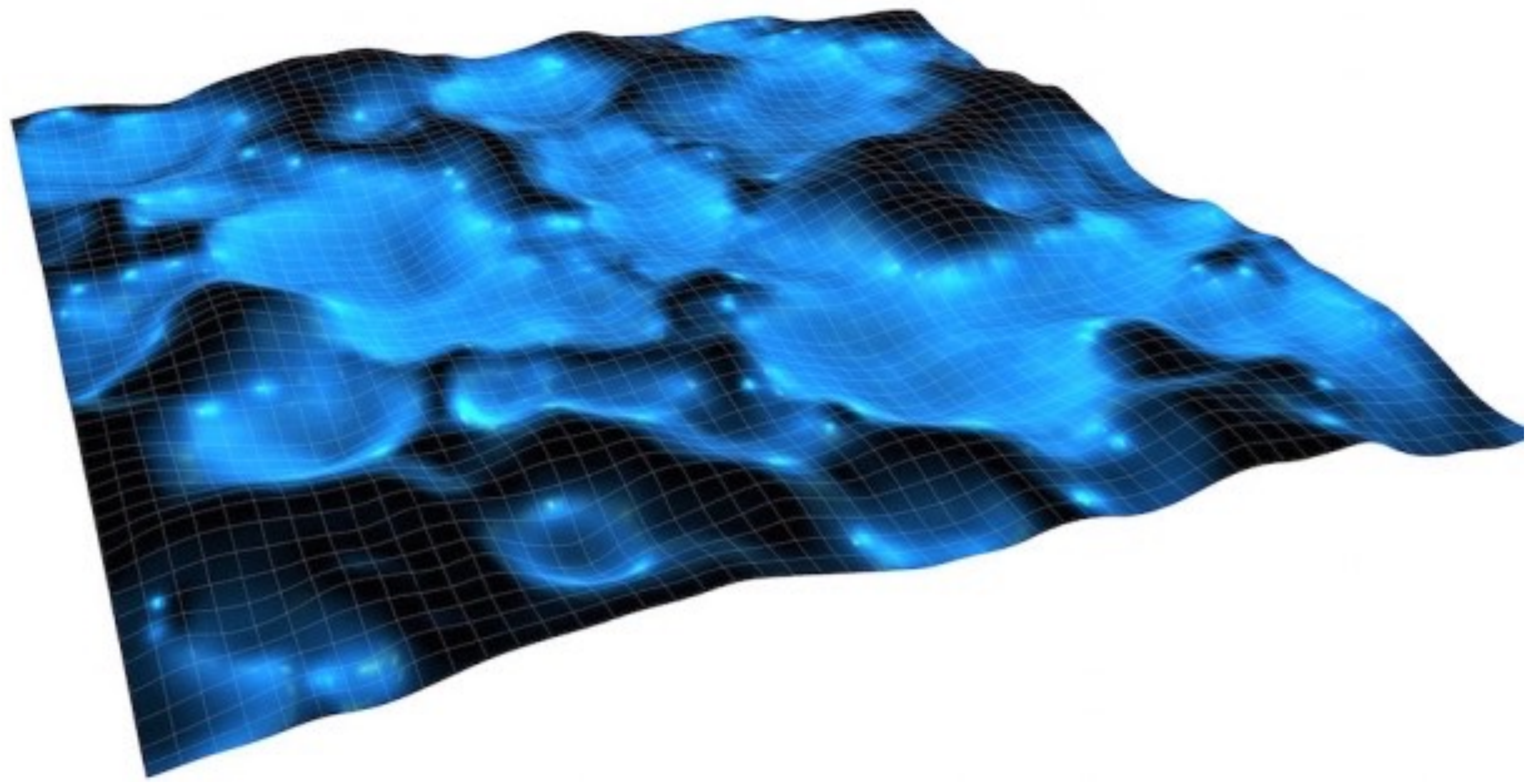


$$E \equiv -u^{\mu} k_{\mu}$$

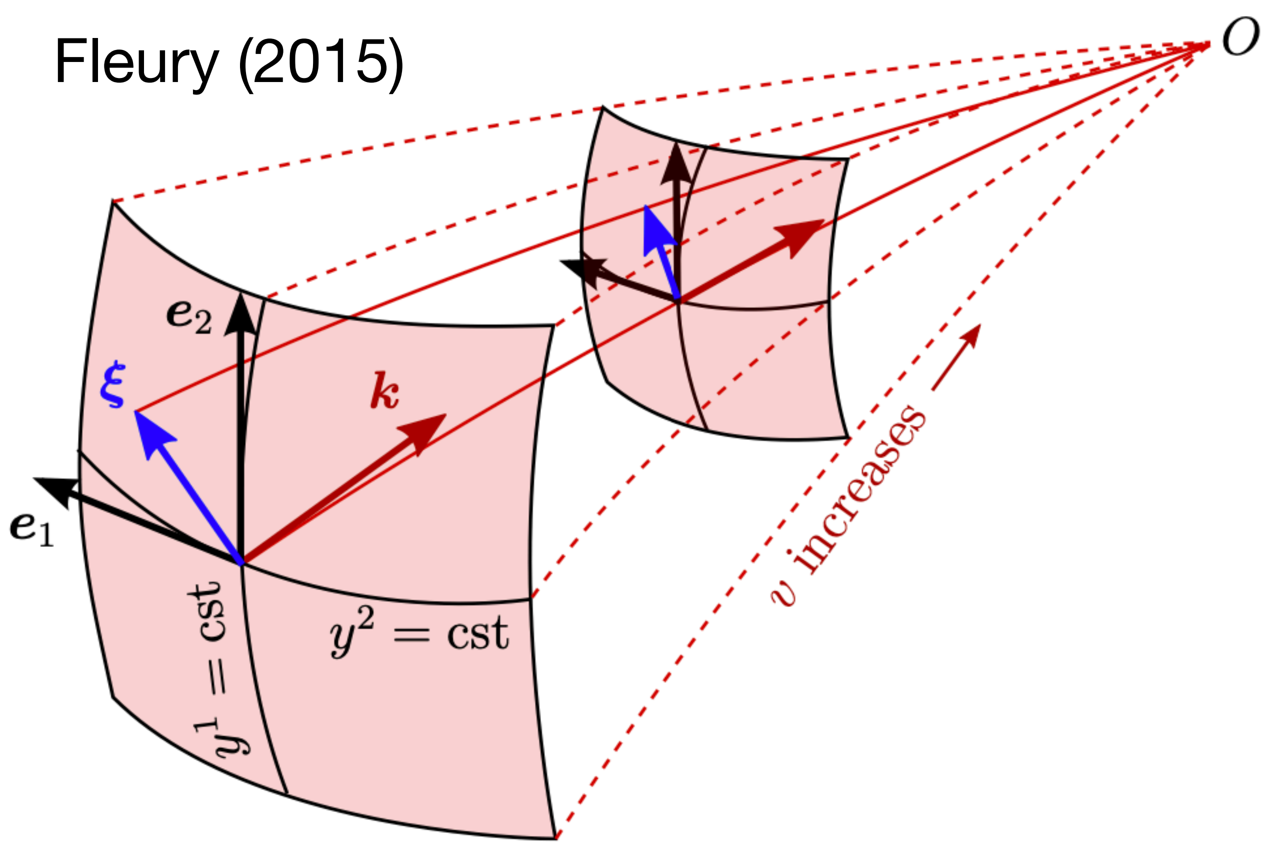
✓ Calculate the energy of the photon as seen by a co-moving observer

$$1 + z \equiv \frac{E_S}{E_O}$$

✓ Calculate the observed redshift



Fleury (2015)



✓ Define an infinitesimal beam of light rays, track their separation along geodesic

(skipping a lot…)

$$\frac{d^2}{ds^2}\mathcal{D}^A{}_B = \mathcal{R}^A{}_C\mathcal{D}^C{}_B$$

✓ Evolve Jacobi matrix equation along the geodesic

Jacobi matrix relates physical attributes of source to how its observed

$$\det(E\,\mathcal{D}^A{}_B) = \frac{A_S}{\Omega_O} \equiv D_A^2$$

$$D_L = D_A(1+z)^2$$

✓ Its determinant is the angular diameter distance

Optical tidal matrix

$$\mathcal{R}_{AB} = \begin{pmatrix} \mathscr{R} & 0 \\ 0 & \mathscr{R} \end{pmatrix} + \begin{pmatrix} -\mathrm{Re}(\mathscr{W}) & \mathrm{Im}(\mathscr{W}) \\ \mathrm{Im}(\mathscr{W}) & \mathrm{Re}(\mathscr{W}) \end{pmatrix}$$

$$\mathscr{R} \equiv -\frac{1}{2}R_{\mu\nu}k^\mu k^\nu$$

$$\mathscr{W} \equiv -\frac{1}{2}C_{\mu\nu\alpha\beta}\sigma^\mu k^\nu k^\alpha \sigma^\beta$$

We can calculate the angular diameter distance and redshift along a geodesic from the simulation

Fluctuations in these quantities are caused by the total convergence:

$$\kappa = \kappa_g + \kappa_v + \kappa_{\rm sw} + \kappa_{\rm isw}$$

(within PT with scalar only contributions)

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + (1-2\Phi)d\boldsymbol{x}^2]$$

In general, we can get this total convergence as fluctuations in dA from our ray tracing:

$$\kappa = \frac{D_A - \bar{D}_A}{\bar{D}_A}$$

(e.g. Fleury 2015)

Gravitational lensing - should dominate at z > 0.5 or so

$$\kappa_g = \int_0^{\chi_s} d\chi (\chi_s - \chi) \frac{\chi}{\chi_s} \nabla_\perp^2 \Phi$$

$$\approx \frac{3}{2} H_0^2 \Omega_m \int_0^{\chi_s} d\chi (\chi_s - \chi) \frac{\chi}{\chi_s} [1 + z(\chi)] \delta,$$

Approximations here:

1. Poisson eq
2. Sub-horizon implies derivatives of phi are small

Doppler lensing

(my observers are co-moving with fluid, shouldn't be contributing)

$$\kappa_v = \frac{1+z_s}{H\chi_s} \boldsymbol{v}_o \cdot \boldsymbol{n} + \left(1 - \frac{1+z_s}{H\chi_s}\right) \boldsymbol{v}_s \cdot \boldsymbol{n},$$

Sachs-Wolfe

$$\kappa_{\rm sw} = 2\Phi_s - \Phi_o + \frac{1+z_s}{H\chi_s}(\Phi_o - \Phi_s),$$

Integrated Sachs-Wolfe

(Second term is zero for EdS)

$$\kappa_{\rm isw} = -\frac{2}{\chi_s} \int_0^{\chi_s} d\chi\, \Phi + 2\left(1 - \frac{1+z_s}{H\chi_s}\right) \int_0^{\chi_s} d\chi\, \Phi',$$

Bacon+ (2014)

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We have the density field along each line of sight

Use this along with the "background" quantities consistent with large-scale averages of the simulation (EdS to within <1%)