Self-similar, Weak Shock Propagation with Accretion

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Outline

• Self-similarity?
  • Sedov-Taylor blastwave
  • New self-similar solutions
• Application to failed supernovae
• Summary/Conclusions
• Questions
Self-similarity

- Mass, momentum, energy must be conserved across this shock. Yields jump conditions:

\[ v[r_{sh}(t)] \propto v_{sh}, \quad \rho[r_{sh}(t)] \propto \rho_a, \quad p[r_{sh}(t)] \propto \rho_a v_{sh}^2 \]

- These boundary conditions occur at a time-dependent location, but can remove this if we introduce \( \xi \equiv r/r_{sh}(t) \) in place of Eulerian radius

- Assume solutions to fluid equations of the form

\[ v = v_{sh}(t)f(\xi), \quad \rho = \rho_a g(\xi), \quad p = \rho_a v_{sh}(t)^2 h(\xi) \]

- This works if \( r_{sh}^\alpha v_{sh}^2 = \text{const} \)

- If energy conserved,

\[ E = \int_0^{r_{sh}} \frac{1}{2} \rho v^2 r^2 dr = \frac{1}{2} r_{sh}(t)^3 v_{sh}(t)^2 \rho_a \int_0^1 \xi^2 f(\xi)g(\xi)d\xi \quad \Leftrightarrow \quad \alpha = 3 \Rightarrow r_{sh} \propto t^{2/5} \]
New Self-similar Solutions

• Recall assumptions of ST:
  • Strong shock — neglect ambient sound speed
    • Kinetic energy >> Thermal energy of ambient gas
  • No gravity
    • Kinetic energy >> Grav energy
  • What happens when these are not satisfied?
    • Does Sedov-Taylor (~ energy-conserving) still describe shock propagation?
New Self-similar Solutions

• In general, probably not:
  • In grav. field, sweeping up ambient material adds binding energy
    • Total energy behind shock not conserved
    • Jump conditions depend on Mach number
      • Adds additional spatial dependence
      • Gravity adds additional timescale/lengthscale
New Self-similar Solutions

• But:
  • If ambient gas in HSE with *point mass*, and adiabatic: $\rho = r^{-n}$, $p = \frac{1}{n + 1} \frac{GM}{r} r^{-n}$, $c_s \simeq \sqrt{\frac{GM}{r}}$
  • If kinetic energy $\sim$ grav potential energy, $\frac{1}{2} v^2 \simeq \frac{GM}{r} \Rightarrow v \simeq \sqrt{\frac{GM}{r}}$
  • And if we let
    $v_{sh} = V \sqrt{\frac{GM}{r_{sh}}} \Rightarrow r_{sh} = \left( \frac{3}{2} V \sqrt{GMt} \right)^{2/3}$, $v = v_{sh} f(\xi)$, $\rho = \rho_a(r_{sh}) g(\xi)$, $p = \rho_a(r_{sh}) v_{sh}^2 h(\xi)$, $\xi = \frac{r}{r_{sh}(t)}$
  • Then
    • Mach $\sim$ constant, boundary conditions satisfied self-similarly
    • Inserting above into fluid equations gives three ODEs
    • Can numerically integrate to find solutions; importantly depend on $V!$
New Self-similar Solutions

• What sets shock velocity $V$?
  • For Sedov-Taylor, $V$ determined from energy constraint
  • Here, however, energy behind shock not conserved…

• Can show that there is a sonic point in these self-sim sols
  • In order for quantities to smoothly pass through, need special value of $V$
Application to Failed Supernovae

• One potential application = failed supernova:
  • Massive star, core collapse
  • Protoneutron star forms, bounces, launches shock
  • If shock stalls and cannot be revived:
    • Continued accretion forms black hole
    • Star is accreted by black hole
    • Disappearing star…
Application to Failed Supernovae

• …But that’s not all:

• During neutron star formation
  • Ton of neutrinos radiated, \( \sim few \times 0.1 M_\odot \)
  • Reduces gravitational field

\[ f_e \approx 0.1 \, M_\odot \]
Application to Failed Supernovae

• While stalled shock is sitting there
  • Overlying envelope (still in ~ HSE) responds to changing gravitational field
  • Result: weak sound pulse generated in interior of star
    ★ Nadyozhin 1980; Lovegrove & Woosley 2013
  • Pulse steepens as it goes down density gradient
Failed Supernovae

• For supergiant:
  • Pulse steepens into weak shock (Mach ~ 1) near base of hydrogen envelope
  • Shock propagates through ~2-3 decades in radius

• Importantly, this all happens while the shock stalls, then fails, then creates a black hole and results in accretion

• Thus, have outward-propagating ~ weak shock, accretion at center, so conditions seem right…
Application to Failed Supernovae

• Fernandez+(2018) ran simulations (FLASH) of failed supernovae for RSGs, BSGs, YSGs, WRs (MESA)

• Focus on specific case of $22 M_\odot$ YSG

• Why?
\[ \rho \propto r^{-2.5} \]

Inner edge of hydrogen envelope
The graph shows the relationship between velocity ($v$) and radius ($r$) for two different power laws:

1. $v \propto r^{-1/2}$
2. $v \propto r^{-1/4}$

The graph plots $v$ in units of cm s$^{-1}$ against $r$ in units of cm, with a range from $5 \times 10^{11}$ to $1 \times 10^{13}$ cm for $r$, and from $0$ to $4 \times 10^7$ cm s$^{-1}$ for $v$. The dotted line represents $v \propto r^{-1/4}$, while the dashed line represents $v \propto r^{-1/2}$. The graph illustrates how velocity decreases with increasing radius, following these power laws.
\[ \dot{M} \propto t^{-2/3} \]

- Numerical
- Freefall
- Self-similar
Conclusions

• Actually seems to work!
  • Predicts propagation of shock
  • Predicts time and space-dependent velocity, density, pressure
  • Predicts accretion rate
  • Predicts amount of ejecta
• If we see one of these things (and the accretion generates a luminous outburst)
  • Maybe we can use these to predict stellar properties?
  • Black hole properties?