Detecting Diffractive Lensing in Astrophysical Gravitational Waves

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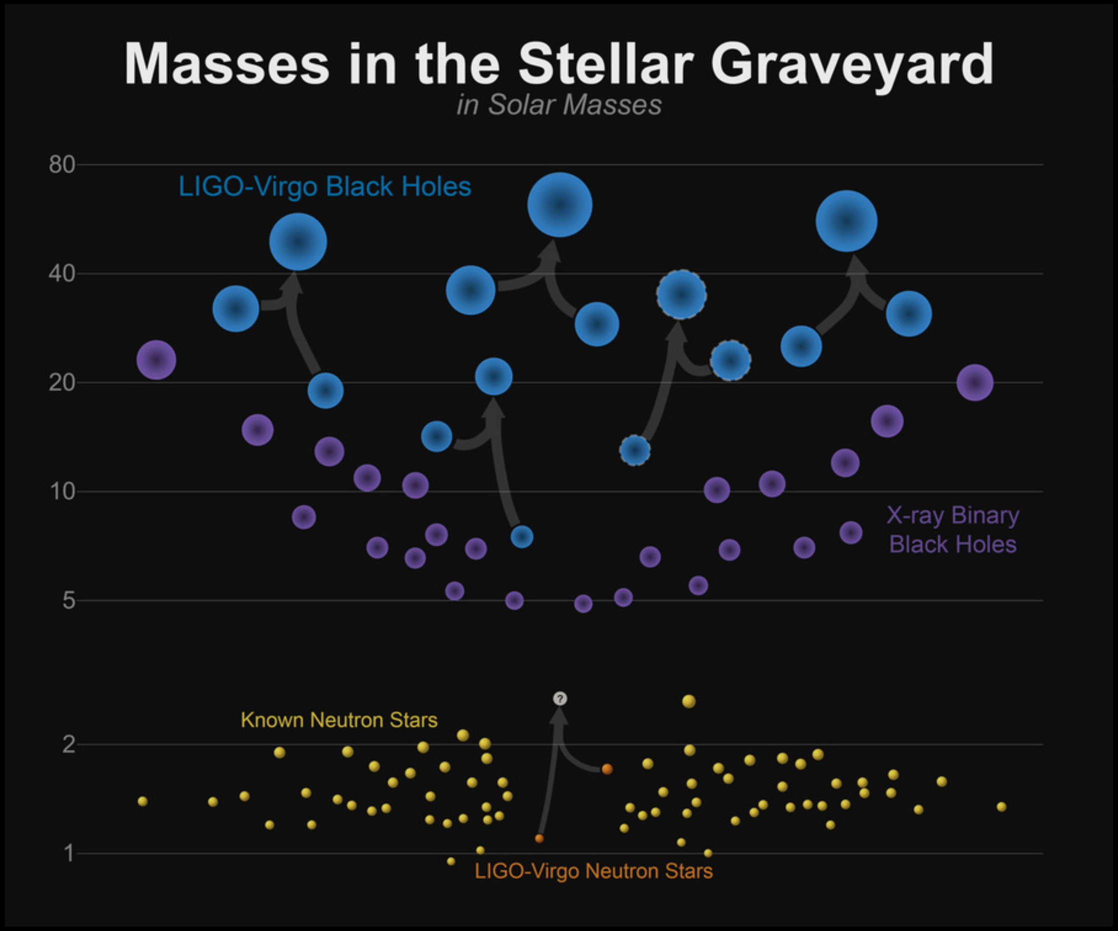
Gravitational Waves: A New Window Into the Universe



LIGO Hanford/Livingston

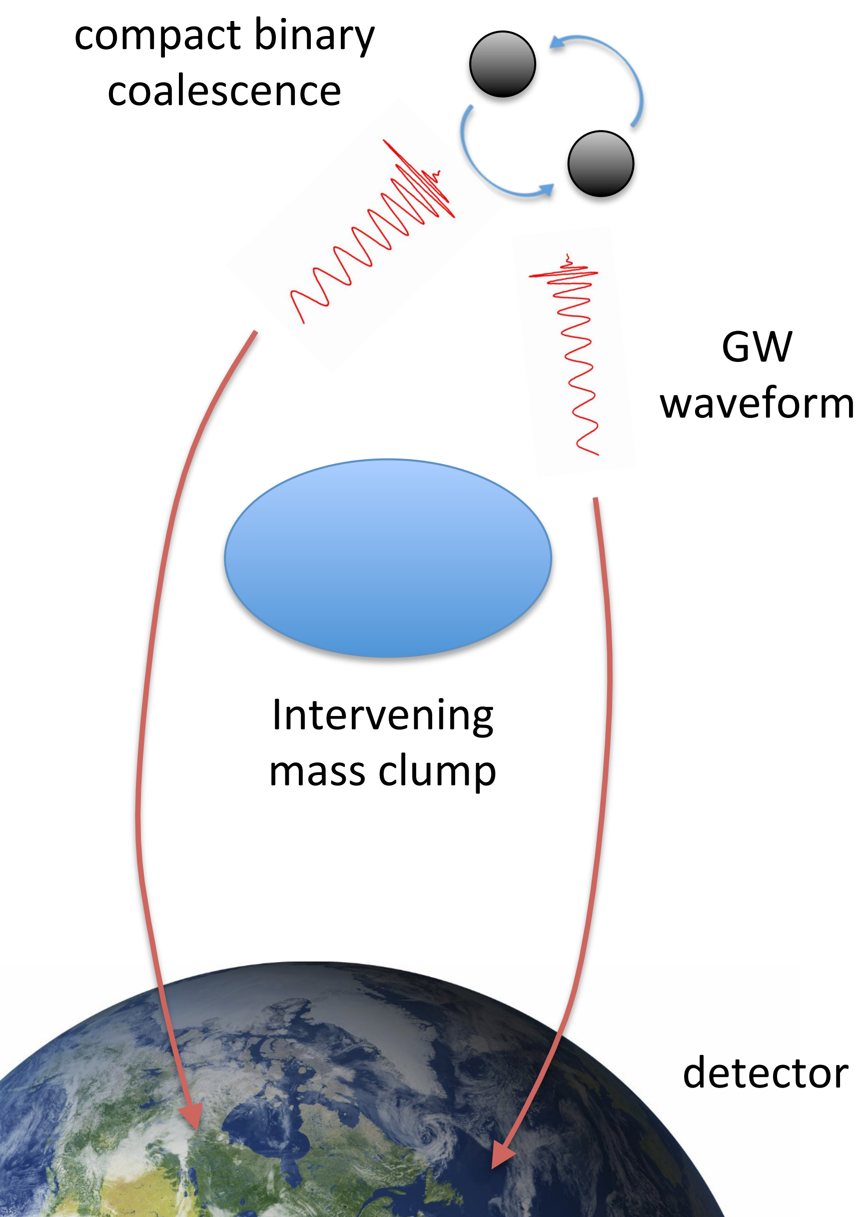
Virgo

KAGRA

Space-based observatory (LISA)
Third generation detector

LIGO-India

Gravitational lensing of gravitational waves



Wang, Stebbins & Turner 96'
Li, Mao, Zhao & Lu 18'

- Regime of geometrical lensing

$$\lambda_{GW} \ll R_{de}$$

- Amplification of strain

$$h' = \sqrt{\mu}\, h$$

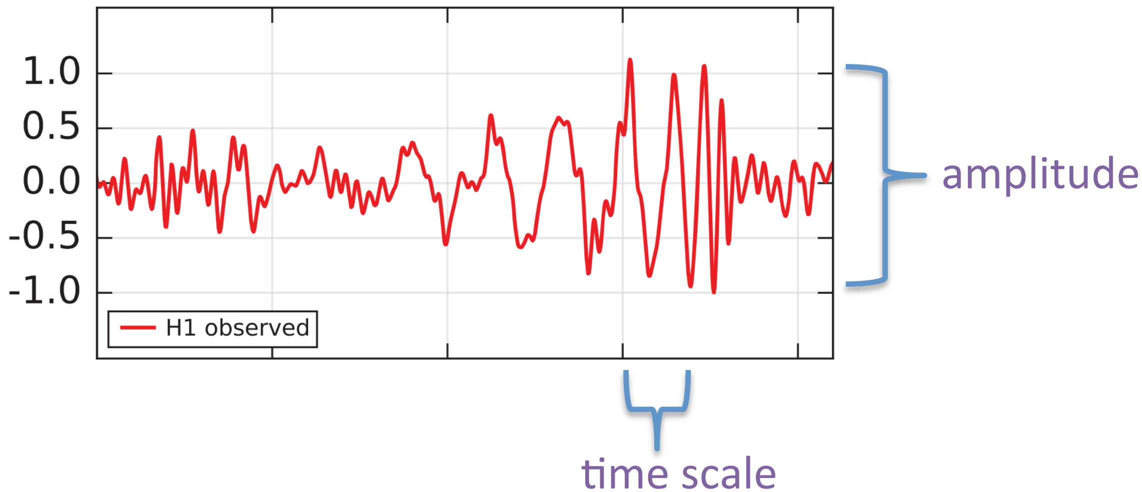
- Wave frequency unchanged

Observational difficulty with geometrical lensing

Dai, Venumadhav & Sigurdson 2017
Ng, Wang, Broadhurst & Li 2017
Broadhurst, Diego & Smoot 2018
Oguri 2018

Apparent mass scale **M'**
Apparent source redshift **z'**

$$M\,(1+z) = M'\,(1+z') \qquad \frac{\sqrt{\mu}}{d_L\,(z)} = \frac{1}{d_L\,(z')}$$



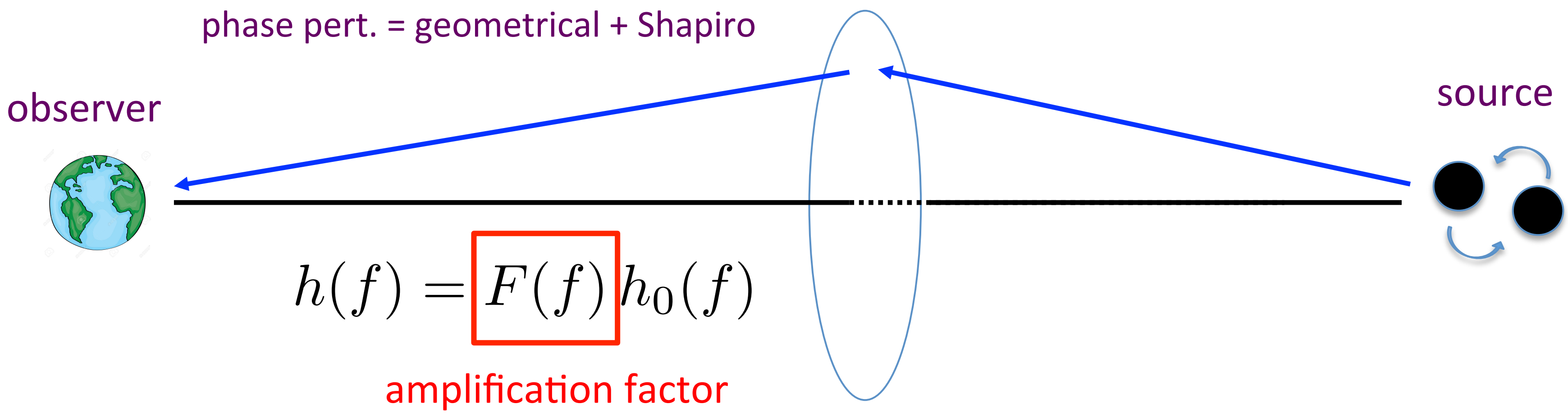
Without EM observations, magnification cannot be recovered from a single lensed image

Topological (Morse) phase shift for **flipped** images;
however degenerate with orbital orientation; Dai & Venumadhav 2017

Multiple lensed images resolvable in time domain
Require fine-tuned impact parameters and large column density
Can be produced by cluster/galaxy lenses; difficult for low-mass lenses

Lensing in wave-diffraction regime

Takahashi & Nakamura 2003
Takahashi PhD thesis



$$h(f) = \boxed{F(f)}\, h_0(f)$$

amplification factor

Limit of geometric optics

$$w = 2\pi\, f\, (1 + z_L)\, G\, M_L / c^2 \gg 1$$

(if only one image) $F_{\rm geo}(f) \longrightarrow \boxed{\sqrt{\mu_{\rm geo}}}\, e^{i\, 2\pi\, f\, (1+z_L)\, \boxed{\tau_{\rm geo}}}$ time delay

(frequency-independent) amplification

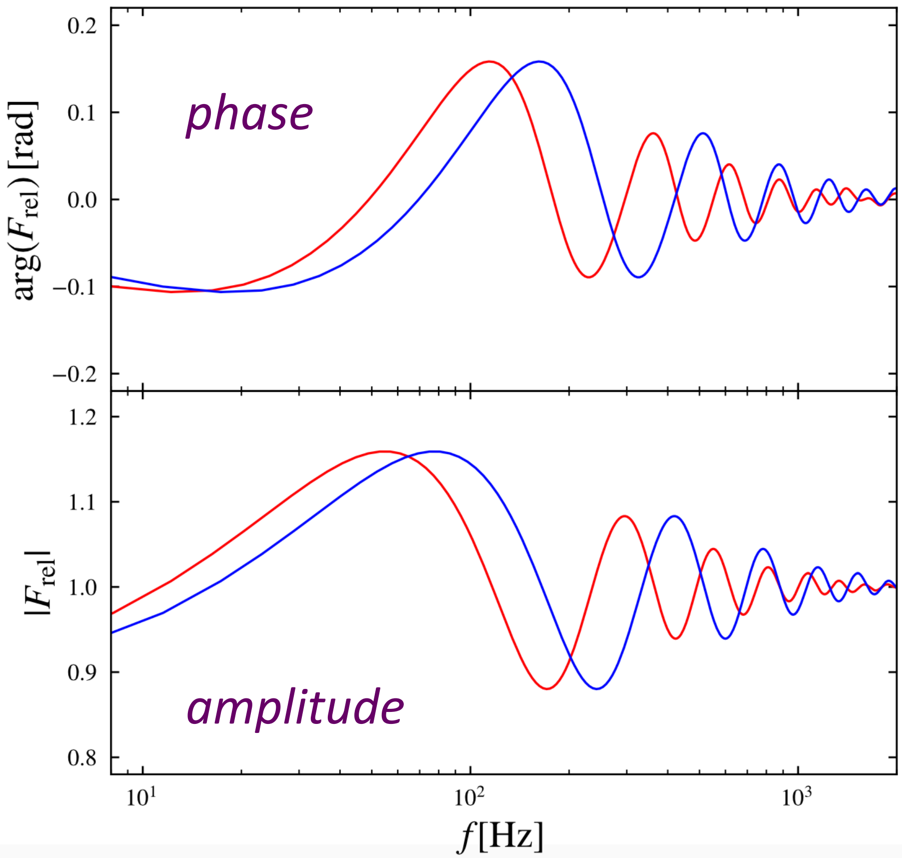
Regime of wave diffraction

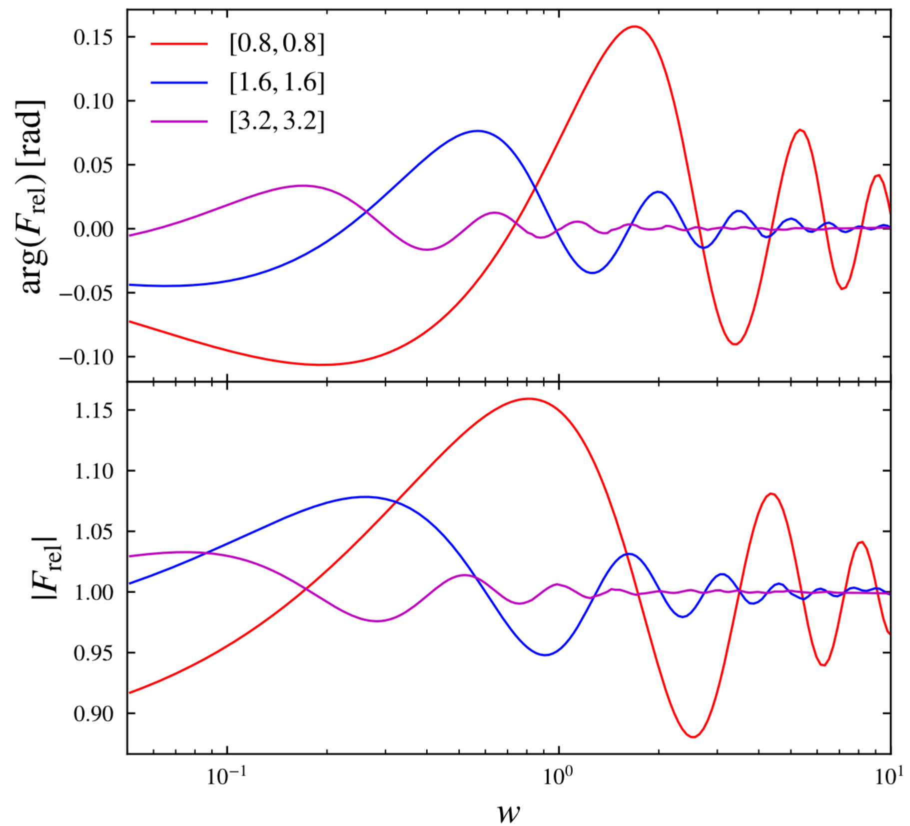
$$w = 2\pi\, f\, (1 + z_L)\, G\, M_L / c^2 \sim \mathcal{O}(1)$$

Define $F_{\rm rel}(f) := F(f) / F_{\rm geo}(f)$ Encode information of waveform distortion !

Amplitude and Phase Modulations

Ground-based band **f ~ 10-1000 Hz** sensitive to (interestingly) small lens masses **M ~ 100-1000 Msun** Dai, Li, Zackay, Mao & Lu 2018





Pseudo-Jaffe lens $\sigma_v \sim 1 - 2\,\mathrm{km/s}$

Impact parameter $\theta_E \sim 4\pi\,(\sigma_v/c)^2$

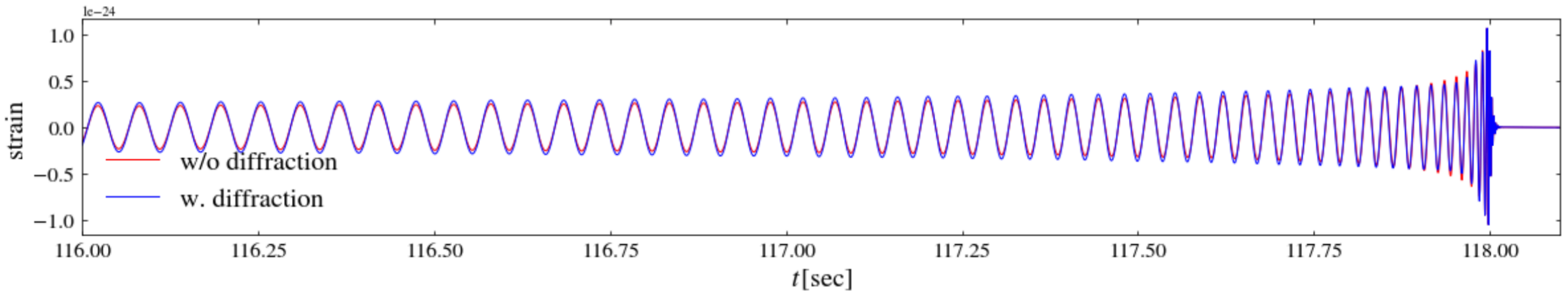
Size of modulation inversely proportional to the impact parameter

Lens mass scale increases

Diffraction signature is subtle

Small modulus and phase perturbations ~ 10-20% or even smaller!

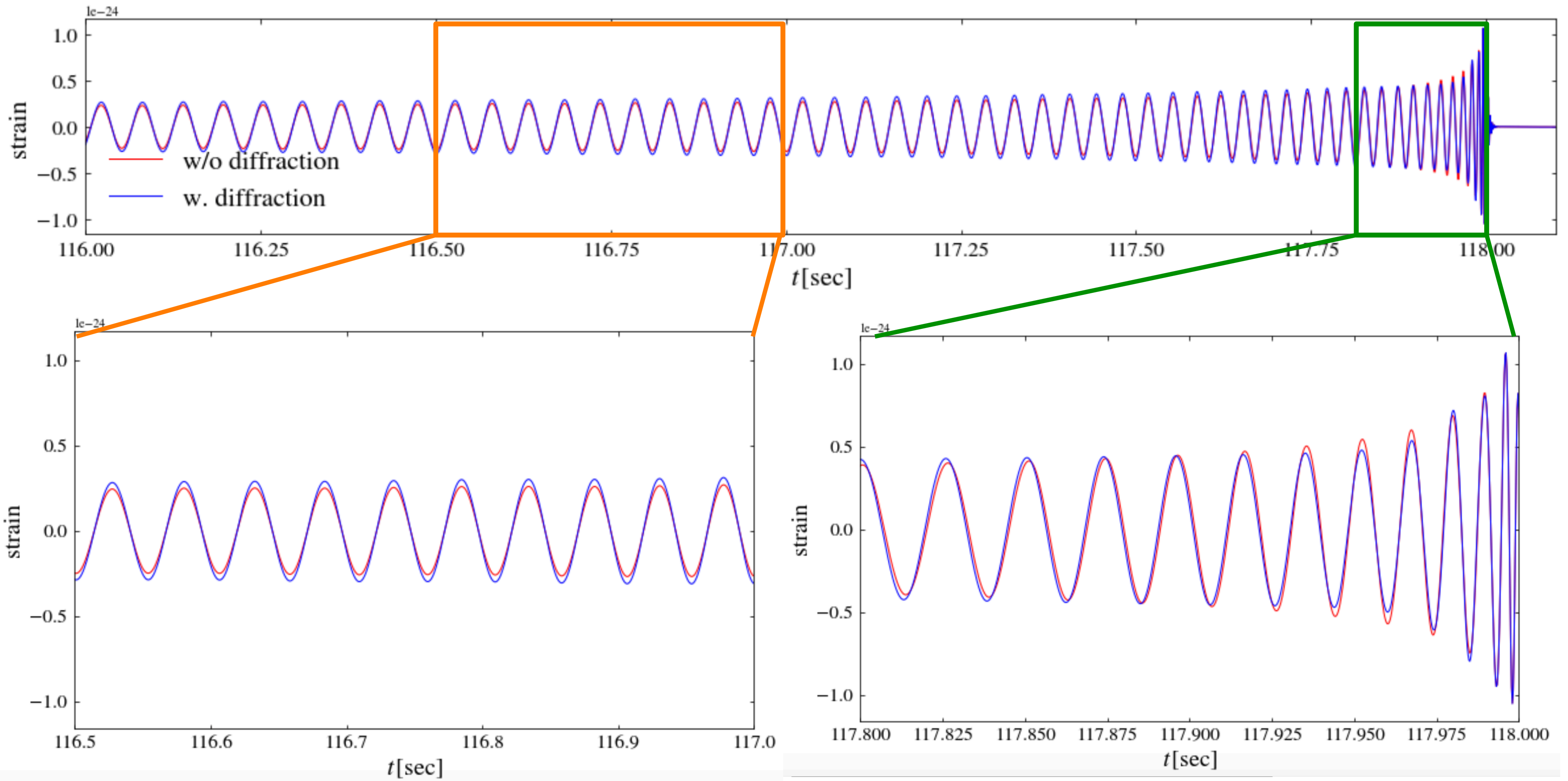
Can you see the amplitude/phase modulations?



Diffraction signature is subtle

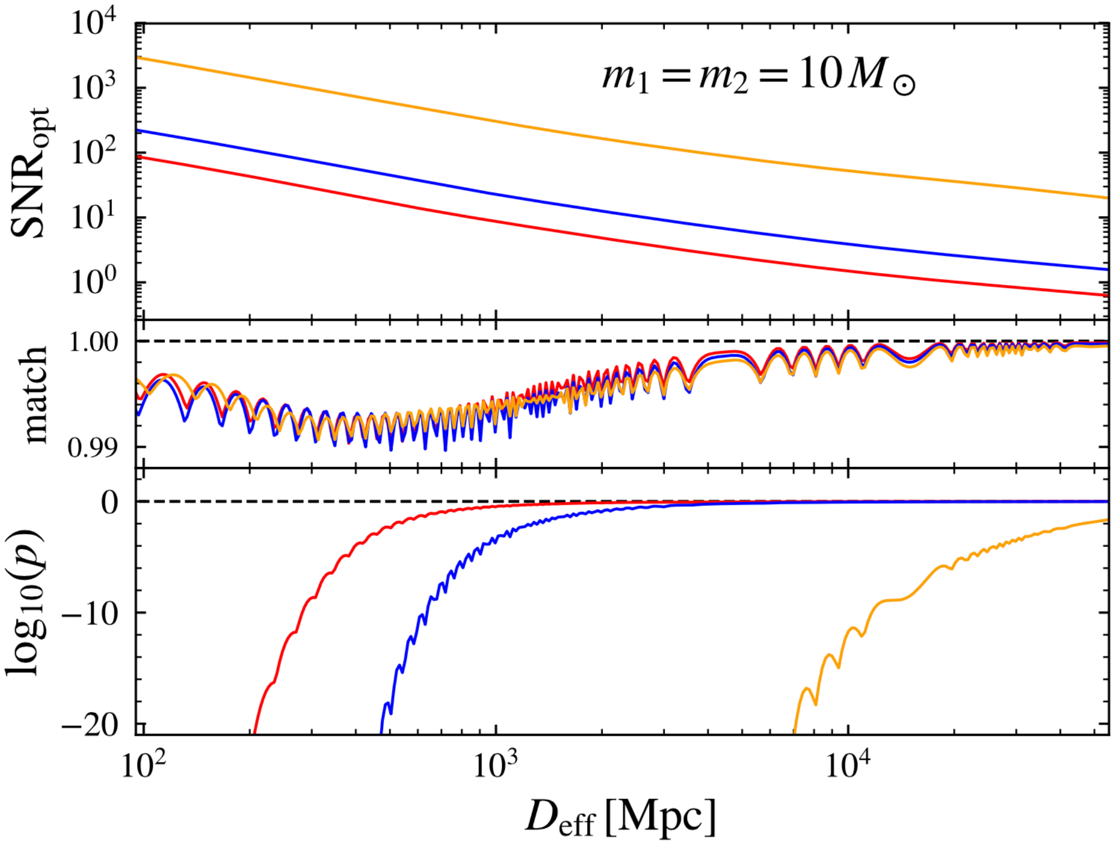
Small modulus and phase perturbations ~ 10-20% or even smaller!

Can you see the amplitude/phase modulations? Hmm … not so impressive …



Match with unlensed templates is (nearly) unaffected.

Diffraction signature still detectable through **the improvement in the likelihood** when **amplitude/phase modulations** are included into the waveform.



aLIGO_MID_LOW

aLIGO_DESIGN

Einstein Telescope

$$\mathrm{match} := \frac{\langle h_L | h_{\mathrm{BF}} \rangle}{\sqrt{\langle h_L | h_L \rangle \langle h_{\mathrm{BF}} | h_{\mathrm{BF}} \rangle}}$$

$$\begin{aligned} \ln p &= -\left(\mathrm{SNR}^2_{\mathrm{opt}} - \mathrm{SNR}^2_{\mathrm{unlen}}\right)/2 \\ &= -\frac{1}{2}\left(\langle h_L | h_L \rangle - \frac{\langle h_L | h_{\mathrm{BF}} \rangle^2}{\langle h_{\mathrm{BF}} | h_{\mathrm{BF}} \rangle}\right) \end{aligned}$$

Matched filtering and some practical difficulties

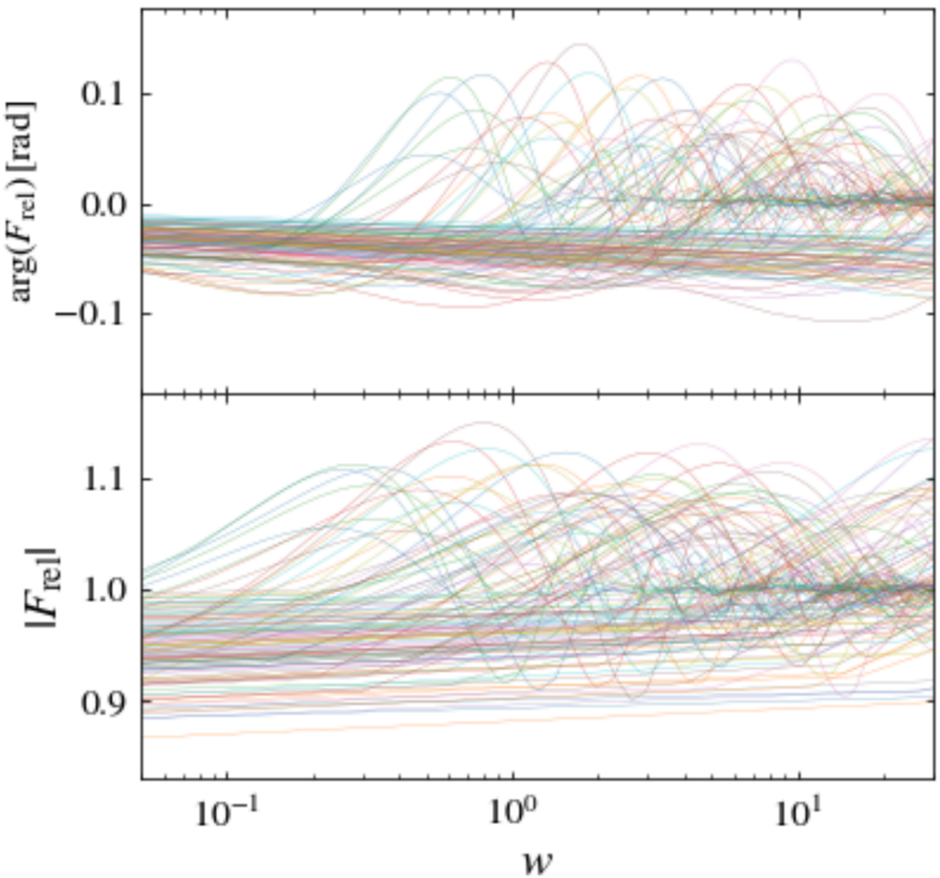
- Matched filtering requires the precise knowledge of F(f)

  e.g. Takahashi & Nakamura 2003, Cao+ 2014, Jung & Shin 2017

- F(f) depends on too many parameters: lens profile, distances, impact parameter, etc.

- The correct lens profile to use is unknown.

- Have to search with a large number of templates. Look-elsewhere effect needs to be quantified.



An agnostic method based on dynamic programming

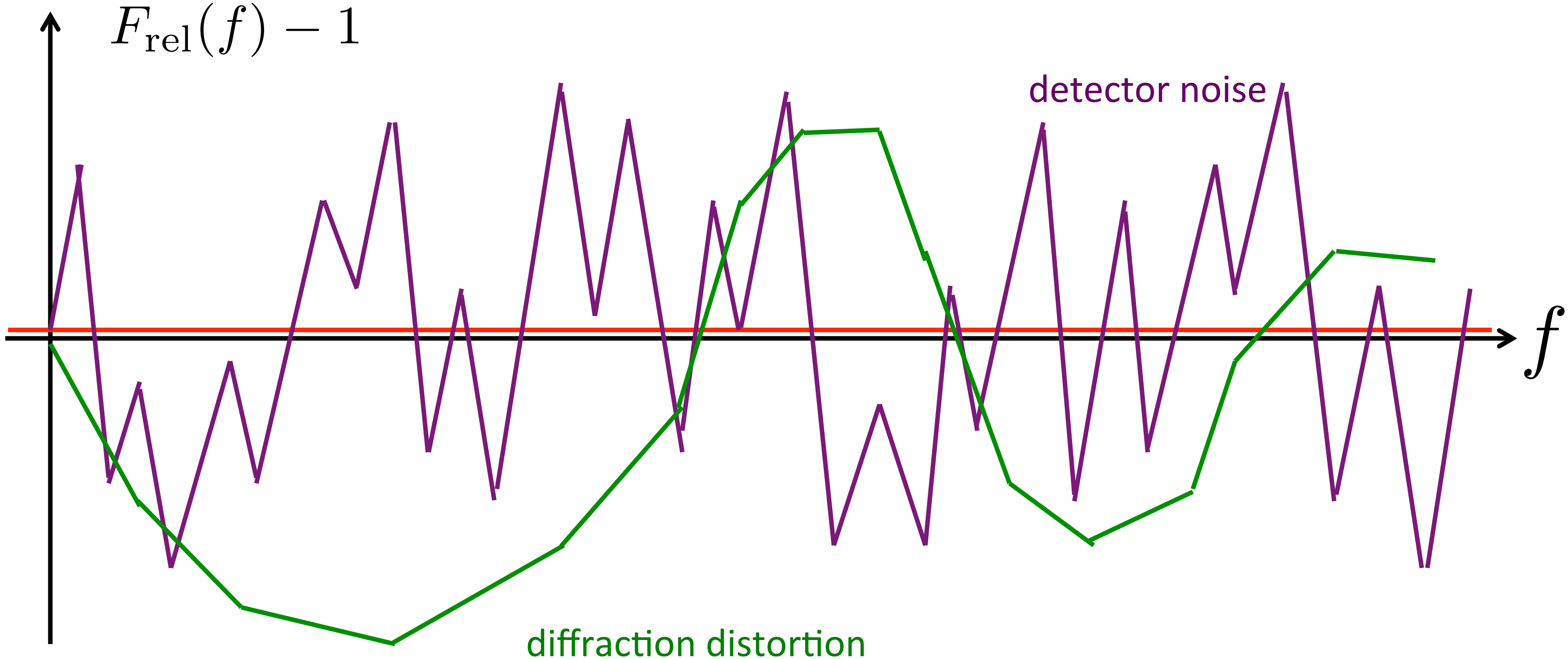
Dai+ 2018

$$\mathcal{S} := \int \mathcal{D}g(f)\, \boxed{\mathcal{P}\left[g(f)\right]} \prod_{a=1}^{N_d} \boxed{\frac{P[s_a(f)|g(f)\, h_{\mathrm{BF},a}(f)]}{P[s_a(f)|h_{\mathrm{BF},a}(f)]}}$$

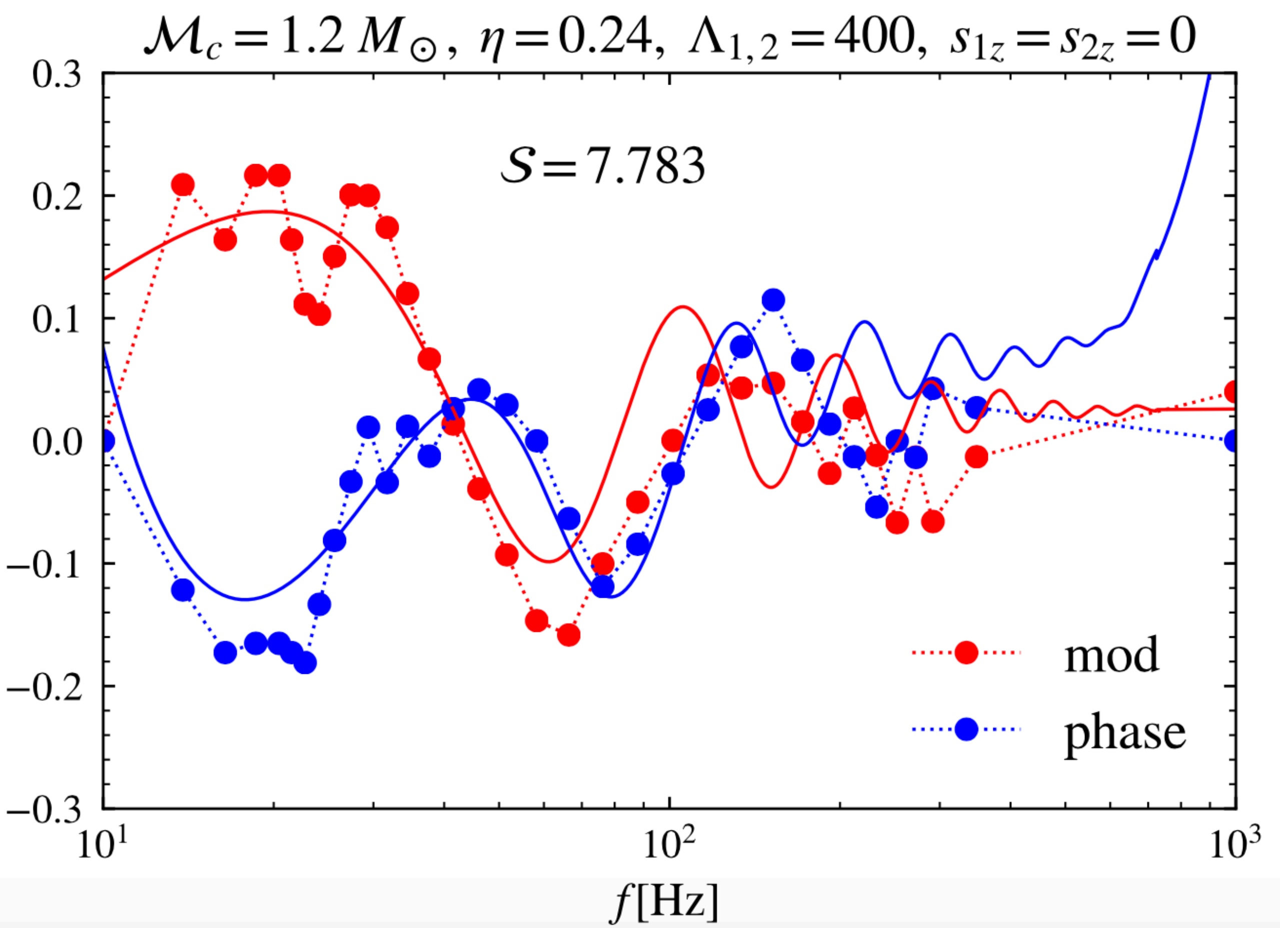
prior (Markovian)

likelihood improvement

$h_{BF}(f)$ is the **best-fit unlensed** waveform



Reconstructing the modulations



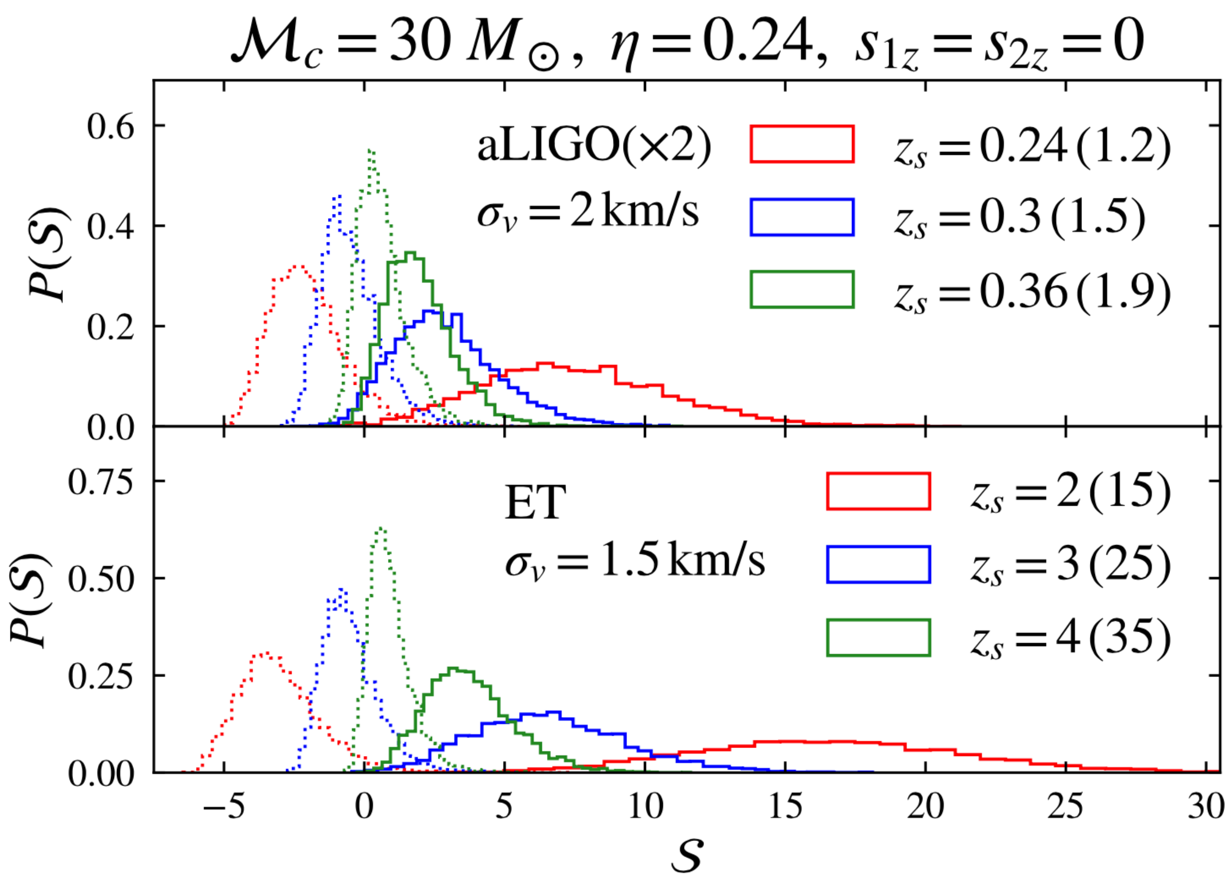
Pseudo-Jaffe lens at z = 0.1

Velocity dispersion σ_v=2 km/s

NS-NS merger at z = 0.2

Can only detect the part of the modulation signal that is not degenerate with source parameters !

Observational prospect: BH mergers are promising!



Assume pseudo-Jaffe halos; mass enclosed within the Einstein radius ~ 100 --- 1000 Msun

aLIGO Hanford/Livingston can probe out to z ~ 0.2 – 0.3.

Further improves **after more detectors join** (Virgo, KAGRA, LIGO-India, etc)

3rd generation detector will be very powerful: z ~ 2 – 4

Discussion

◆ Science case: test CDM theory on sub-galactic scales ?!

- Probe inner region of $M \sim 10^4$—10^6 Msun DM halos
- 3rd gen. detector can use BBHs out to $z \sim 2$-4
- Assume nearly log-flat halo mass function; lensing optical depth ~ few * 10^{-3} if $r_E \sim 1$ pc
- Enough enclosed mass? Small halos show steeper inner profiles than NFW.

  e.g. Dutton & Maccio 2014
- Galaxy lensing events particularly interesting to look at

Thank you!

◆ Degeneracy with **spin-precessing** or **eccentricity** effects?

- Precession can induce amplitude/phase modulations in the frequency-domain waveform. e.g. Apostolatos, Cutler, Sussman & Thorne, 1994; Klein, Cornish & Yunes 2013
- Modulation frequency (~ tens of precession cycles in band) is typically higher than diffraction; amplitude modulation more significant than phase modulation.
- Detailed study would be very valuable; need accurate waveforms.