# Exploring the Physics of Warped Accretion Disks With the Imaging X-ray Polarimetry Explorer 

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## Introduction

The angular momenta of an accretion disk and the compact object (often a black hole) it surrounds are traditionally thought to be aligned. A supernova, however, may kick these out of alignment - it's estimated that $60 \%$ of binaries have misalignments between $5^{\circ}$ and $45^{\circ}$ [3]. Bardeen and Petterson posited that during accretion in a misaligned system, the interplay of disk viscosity and the Lense-Thirring effect will align the disk with the black hole angular momentum [1]. Recently, a General Relativistic Magnetohydrodynamic Simulation found that the disk can warp at a very small radius ( $\mathrm{r}_{\mathrm{BP}} \sim 5 \mathrm{rg}_{\mathrm{g}}$ ), with the inner region aligned with the black hole and the outer region with the binary orbit. [6]. We have developed a general relativistic ray-tracing code that finds the energy spectrum and polarization of a warped disk.


## Ray-Tracing and the Basis of the Tilted Disk

Our ray-tracing code generates photon packages in the accretion disk and tracks them through a Kerr spacetime. It uses the Cash-Karp method to integrate the geodesic equation and parallel transport the polarization vector $f^{\mu}$ :

$$
\frac{d^{2} x^{\mu}}{d \lambda^{2}}=-\Gamma_{\sigma v}^{\mu} \frac{d x^{\sigma}}{d \lambda} \frac{d x^{v}}{d \lambda}
$$

$$
\frac{d x^{\mu}}{d \lambda}=-\Gamma_{\sigma v}^{\mu} f^{\sigma} \frac{d x^{v}}{d \lambda}
$$

When a photon crosses the accretion disk, it scatters using the formalism of [2] for polarized photons scattering off an infinitely thick electron scattering atmosphere. This requires a transformation of the photon from the global Boyer-Lindquist coordinates into the local Lorentz frame of the disk material.
To find this, we define the outer disk as the solutions of the equation

$$
\begin{equation*}
\cos (\theta) \cos (\beta)-\sin (\theta) \cos (\phi) \sin (\beta)=0 \tag{1}
\end{equation*}
$$

and truncate it at 100 r g . We assume the matter in the tilted outer disk moves in circular orbits with a Keplerian angular velocity $\Omega_{\mathrm{K}}=\left(a+r^{3 / 2}\right)^{-1}$, where $a$ is the spin parameter of the black hole. Rotating a circular orbit in the equatorial plane by an angle $\beta$ gives positions
$\theta=\arccos \left(\sin \beta \cos \left(\Omega_{\mathrm{K}} t\right)\right)$
$\phi=\arctan \left(\sec \beta \tan \left(\Omega_{\mathrm{K}} t\right)\right)$.
Now we can find the four basis vectors $\mathbf{e}_{\hat{v}}$ describing the outer disk in terms of the BoyerLindquist basis vectors $\partial_{\mu}$

- $\mathbf{e}_{\hat{0}}$ : The four velocity of the disk material, $\mathbf{u}=d x^{\mu} / d \tau$. This is proportional to $d x^{\mu} / d t$ but is normalized to -1 .
- $\mathrm{e}_{\hat{1}}$ : Proportional to $\partial_{r}$
- $\mathrm{e}_{2}$ : Gradient across the outer disk, pointing up
- $\mathrm{e}_{3}:$ Tangent to the particle orbit (Eq. 2).

All of these are made perpendicular to each other using Gram-Schmidt orthonormalization.

Polarization of the Thermal Emission from a Warped Disk
We simulated an accretion disk with misalignment $\beta=15^{\circ}, \mathrm{r}_{\mathrm{BP}}=8 \mathrm{rg}_{\mathrm{g}}$ around a black hole with $\operatorname{spin} \mathrm{a}=0.9$. We looked at this system with the inner disk inclined at $75^{\circ}$ from eight observers around the black hole in $45^{\circ}$ increments of $\phi$. Here, we'll highlight just two of these observers.


Figure 1: Images of the total emission (left), direct emission (middle), and reflected emission (right) seen by an observer at $\phi=90^{\circ}$ with the outer disk inclined at $75.52^{\circ}$. The color bar gives the surface brightness in logarithmic units. On top of each image, the length of the black bars gives the polarization fraction and their orientation gives the polarization angle.


Figure 2: Energy spectrum (left), polarization fraction (middle), and polarization angle (right) corresponding to Fig. 1. Included are lines for the total emission (solid black) and for aligned disks in the orientation of the outer disk (dashed black) and inner disk (dotted black). The polarization fraction is basically the same as both aligned disks, since the inner and outer disks have the same inclination. The polarization angle, however, tends to match the fully misaligned disk, which is offset from the fully aligned disk by $\sim 15^{\circ}$.


Figure 3: This observer is at $\phi=135^{\circ}$, and the outer disk is inclined at $64.74^{\circ}$. The polarization fraction matches the fully misaligned disk at low energies, but is slightly lower than in the fully aligned disk. At high energies, it is lower than both aligned disks. The polarization angle roughly matches the fully misaligned disk, and is offset from the fully aligned disk by $\sim 12^{\circ}$. This is the visible angle of misalignment.




## Observing Warped Disks with IXPE

- Due to IXPE's energy range ( 2 to 8 keV ), simultaneous observations of sources are necessary to determine their state
- Using the code described here, we are currently working to describe the polarization and time lag of the iron line from a warped disk
- Possible targets that IXPE could observe include:
- Cygnus X-1: Inner disk inclination is $\sim 40^{\circ}[8]$, binary is $(27.1 \pm 0.8)^{\circ}[7]$ - GRO J1655-40: Jet inclination is $(85 \pm 2)^{\circ}[5]$, binary is $(70.2 \pm 1.9)^{\circ}[4]$
- 4U 1957+11: Low mass X-ray binary, consistently in spectrally soft state and well fit by pure thermal spectrum

$\log ($ Energy/keV)


Figure 4: Simulated polarization fraction and angle for Cygnus X-1 (solid black line), given the inclinations of [8] and [7], compared to a completely aligned disk (dotted) and completely misaligned disk (dashed). IXPE could distinguish between the three models to determine if the accretion disk of Cygnus $\mathrm{X}-1$ is indeed misaligned

## References

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